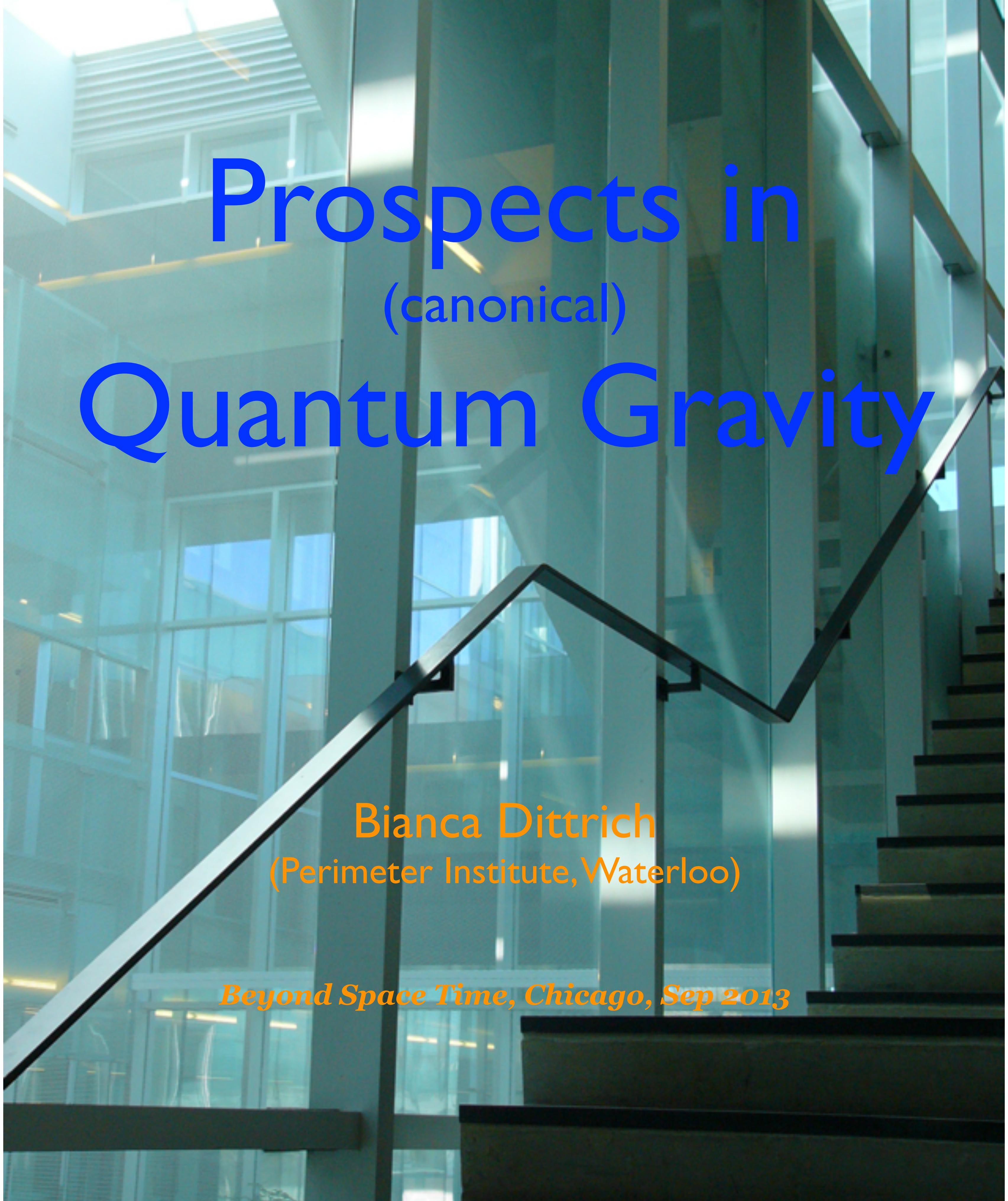


Prospects in
(canonical)
Quantum Gravity

Bianca Dittrich
(Perimeter Institute, Waterloo)

Beyond Space Time, Chicago, Sep 2013

What is observable?

What is vacuum?

What is nothing?

How can we know that there is something?

Canonical Quantization:

Find quantum representation of observable algebra.

$$A \to \hat{A} \quad , \qquad \{A, B\} \to i[\hat{A}, \hat{B}]$$

What are observables?

Problem of time:

- diffeomorphism symmetry (coordinate transformation) is a gauge symmetry of general relativity

- include time translations, which are therefore gauge symmetries

- physical observables should be gauge invariant, hence

  physical observables should be constant in ‘time’

- this argument leads to the the nebulous

  (naive) “problem of time” or “frozen time formalism”

(naive) solution: relational observables [Einstein, Bergmann, Kuchar, Rovelli, BD ...]

“Position of particle (now)” is not an observable.

“Position of particle at 5pm (on Daniele’s clock)” is an observable. This allows a notion of evolution with respect to Daniele’s clock.

There is also a notion of (physical) Hamiltonian that evolves Daniele and the rest of the universe (but not his clock).

So no “problem of time”?

Practical (?) problems:

A. In the canonical formalism observables are pre-- or postdictions.

   Need to solve dynamics of the theory. [But can be done in principle: BD]

B. Good clocks?

Clocks should

better not be too heavy!

Relational observables are widely applied:

[Kuchar, Brown-Kuchar, BD, Tambornino, Giesel, Thiemann, Ashtekar et al, Lewandowski et al, Husain, Pawlowski, ...]

Problem A and Problem B can be avoided if one 'deparametrizes the theory'.

deparametrization = nice (one-parameter family of) gauge fixing = nice system of clocks [BD]

Around flat / homogeneous phase space points: good clocks provided by standard gauge fixings (e.g. longitudinal gauge). These are however highly non-local. [BD, Tambornino]

Allow discussion of fluctuation light cones / causal structure. [BD, Tambornino]

Aether:

If there are no good clocks, lets add them to the system (aether approach) :

-Gaussian reference fluid, harmonic gauge fluid, ... [Kuchar et al]

-(honest) aether [Jacobson]

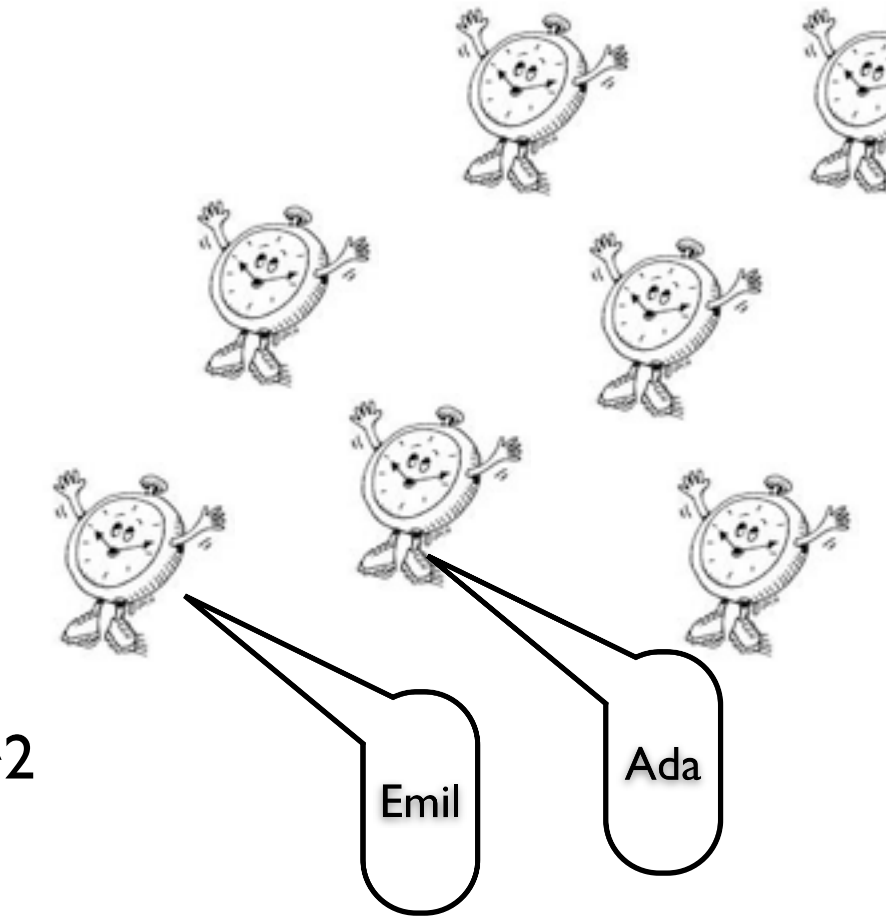
-dust (with different properties) [Brown-Kuchar, Rovelli, Giesel, Thiemann, Husain, Pawlowski,...]

NOAA George E. Marsh A

Allows a "diffeomorphism invariant way" to specify a preferred reference system.

[Gielen, Wise: similar: local Lorentz symmetry]

But what is this aether, dust, ...?



Often very special properties:

... cannot be detected ... not interacting with other matter ...

... peculiar non-relativistic kinetic term

Kinetic energy ~ (Momentum) as opposed to (Momentum)^2

This makes a big difference for properties of observables!

Example: Time of arrival operator. [Aharonov, Unruh: additional uncertainty relation]

$$C = p_t + \frac{p_q^2}{2m}$$

$$F_q(\tau) \ = \ q + \frac{p_q}{m}(\tau - t)$$

$$F_q(\rho) = t + \frac{m}{p_q}(\rho - q)$$

Two-point function of scalar field relative to (four) clock scalars

[BD, Tambornino]

$$\{\phi(\Psi), \phi(\Psi+\epsilon)\} = G(\Psi, \Psi+\epsilon)\left(1+\frac{\mathrm{Energy}(\phi)}{\mathrm{Energy}(\Psi)}\right)$$

encode 'free' dynamics

Green's function on fixed background

Resolution limit for degrees of freedom points depending on energy of clocks.

Similar: for two-point function in path integral approach [Giddings, Hartle, Marolf]

Forming of black holes leads to (super) holographic bound on number of dof's [Giddings, Hartle, Marolf]

Existence of black holes: no perfect clocks?

[Bojowald, Hoehn, Tsobanjan]: Fashionables: change clocks when necessary. Leads to effects similar to non-unitary evolution.

Relational observables

[Kuchar, Brown-Kuchar, BD, Tambornino, Giesel, Thiemann, Ashtekar et al, Lewandowski et al, Husain, Pawlowski, ...]

•solve the problem of time in many situations

•are used successfully, in particular for quantum cosmology [Ashtekar, Agullo, Singh, ...]

•the foundational question remains: What can we measure if there is no space time?

•canonical formalism allows (forces you to think about) to discuss this question

• aether theories [Jacobson, Horava, ...] vs emergence of and space time

What is vacuum?

QFT on (Minkowski) space time background

Vacuum is a highly complicated object, that might be argued to encode

- is of lowest energy ?

- invariant under symmetries of background space time ?

- that might be argued to encode geometry of Minkowski space ?

  [Tomita,-Takesaki; Bisognano, Wichman, ...]

- is cyclic (Fock space generated from vacuum)

Vacuum for quantum gravity?

Kinematical vacuum for loop quantum gravity

$\boxed{\text{the vacuum}}$ $|\mathrm{vac}>$

In LQG vacuum is a maximally squeezed state:

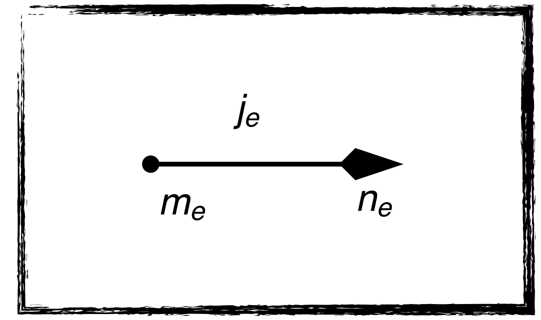
-expectation values and fluctuations of (spatial) geometric observables vanish

-state of no (spatial) geometry

Building up space:

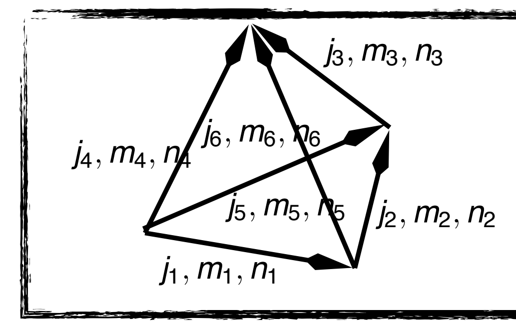
an excited state

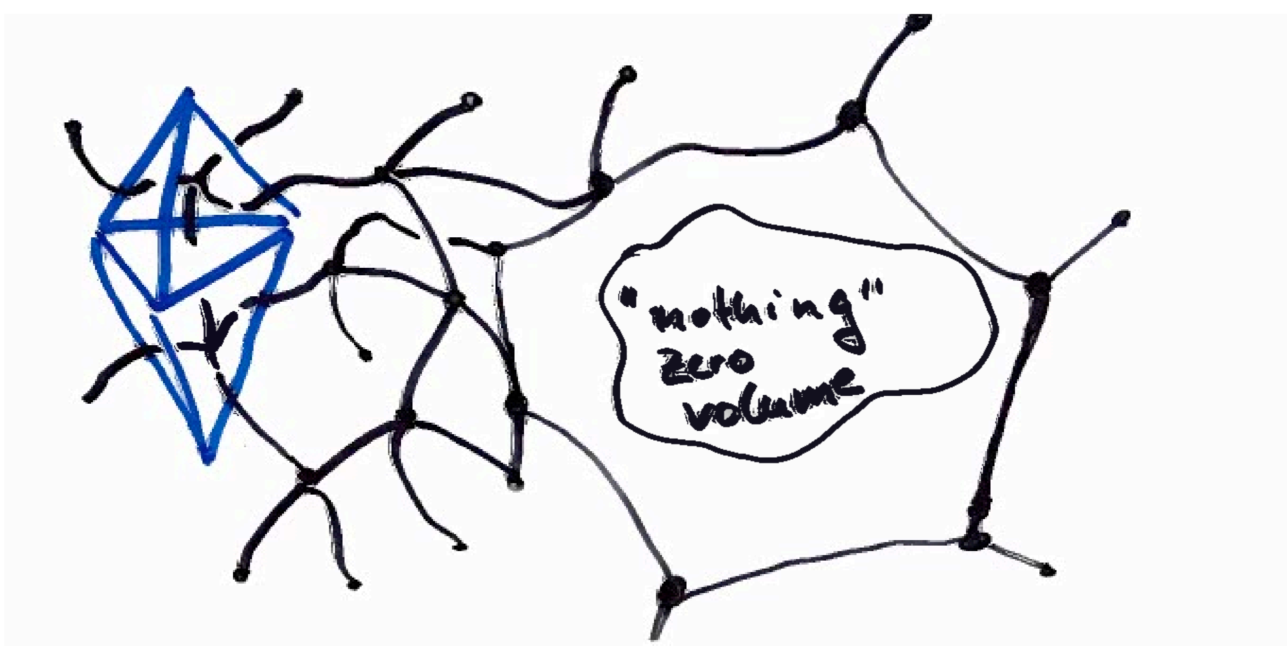
$$\psi_e = \widehat{h(e)}^{j}_{mn}|\mathrm{vac}>$$



an even more excited state

$$\psi_\Gamma = \widehat{h(e)}^{j_1}_{m_1 n_1} \cdots \widehat{h(e)}^{j_N}_{m_N n_N}|\mathrm{vac}>$$





As in Fock space we can create a highly excited state from the vacuum by applying lots of 'creation operators', based on graphs.

Uniqueness result (compare with qft)

LQG provides a unique representation of kinematical observable algebra, that is

-cyclic

-irreducible

-carries a representation of the spatial diffeomorphisms

[F-LOST: Lewandowski, Okolow, Sahlmann, Thiemann - Fleischhack]

Graph (string) like excitation:

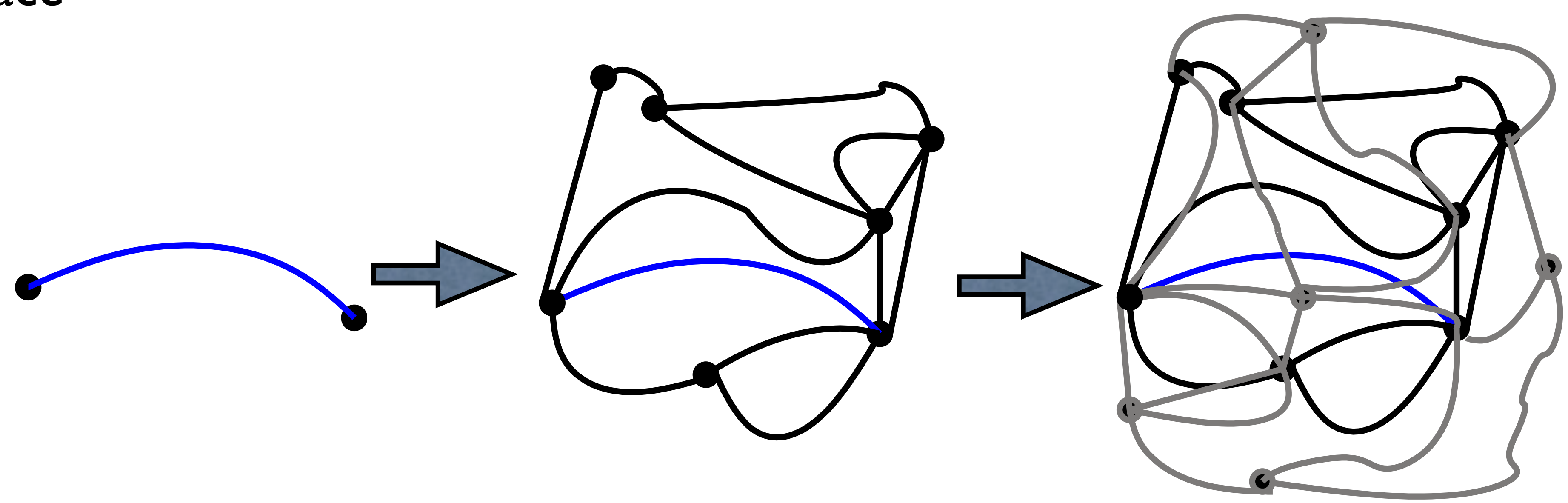
A state on a given graph can be interpreted as a discrete quantum geometry.

Yet LQG is a continuum theory.

Cylindrical consistency in LQG

- family of (kinematical) Hilbert spaces labelled by graphs
- embed states based on graphs into continuum theory (infinitely refined graph)

- discrete states are elements of (equivalence classes) of the continuum Hilbert space



- same state (based on one edge) represented in different Hilbert spaces

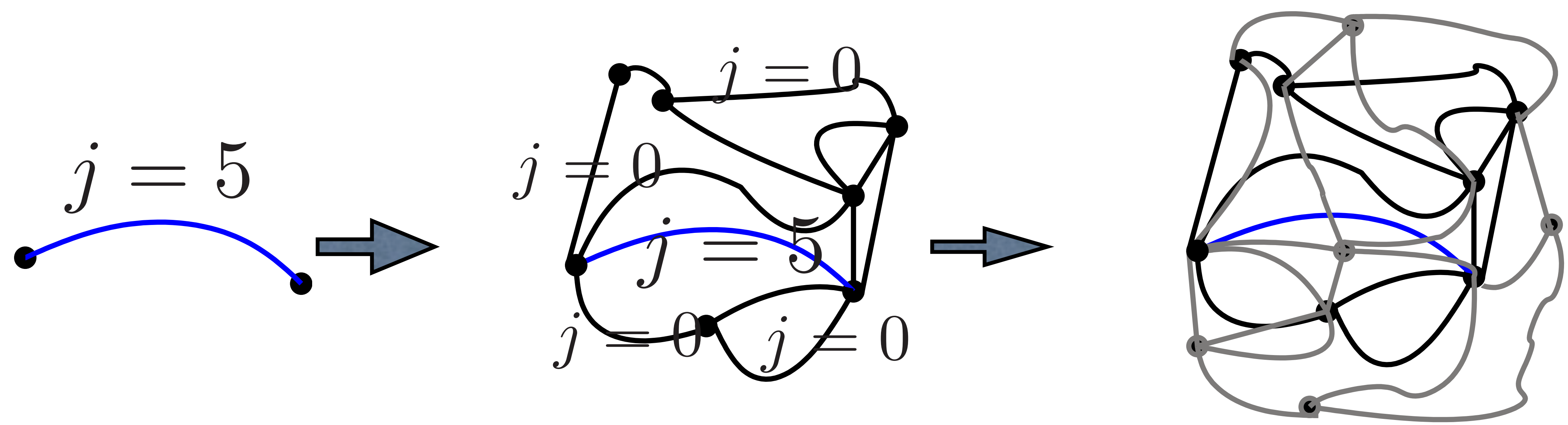
- cylindrical consistency: result of a computation (inner product, expectation value) does not depend which representative one chooses

➡result is valid for continuum theory

- AL-measure: cylindrical consistency for kinematical inner product

[Ashtekar, Isham, Lewandowski 92]

How to embed?



- same state (based on one edge) represented in different Hilbert spaces

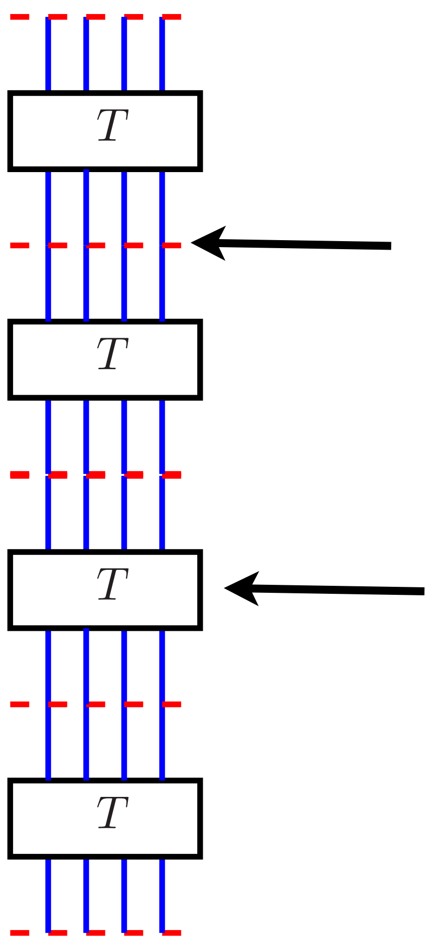
We use the (kinematical) vacuum to 'fill the voids'.

But it gives zero volume to all these voids.

Can we find a better embedding?

➡embeddings (vacuum) should be determined by the dynamics of the theory
➡fine and coarse graining should play a role

How to get vacuum of a statistical system

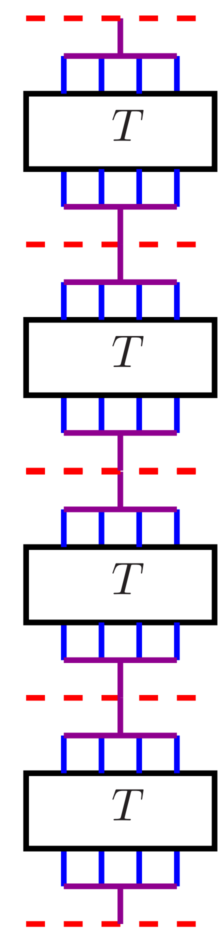


Hilbert space
(with four sites)

transfer operator

$T = \exp(-H)$

$$T^N \underset{N\to\infty}{\longrightarrow}$$ **projector on** **ground state**

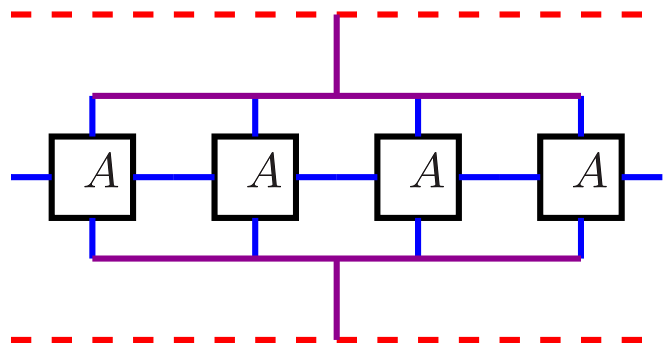


Truncate by restricting $\sum_{\text{ONB}}$ to the eigenvectors of T with the χ largest (in mod) eigenvalues.

But: explicit diagonalization of T difficult.

Dynamically determined embedding maps

[BD 2012]



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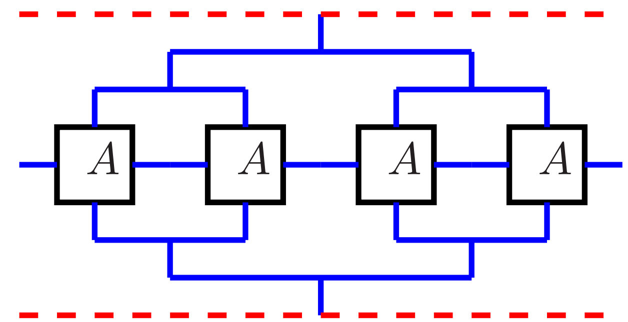
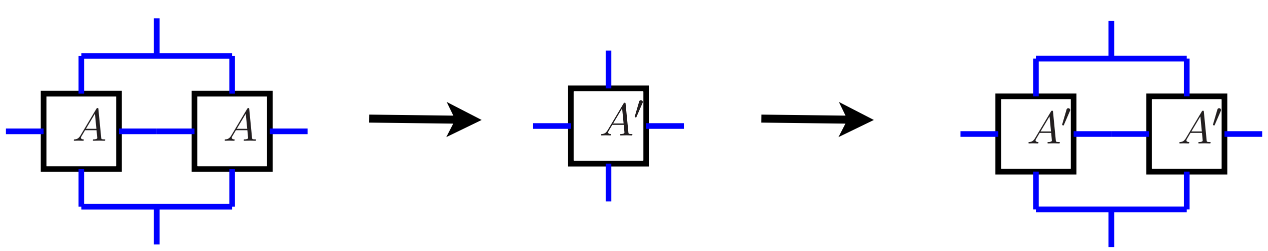
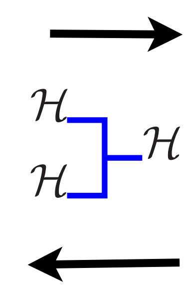


Localize truncations, diagonalize only subparts of transfer operator

iteration procedure



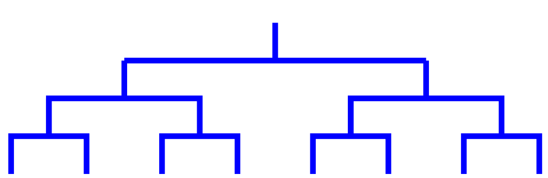
blocking



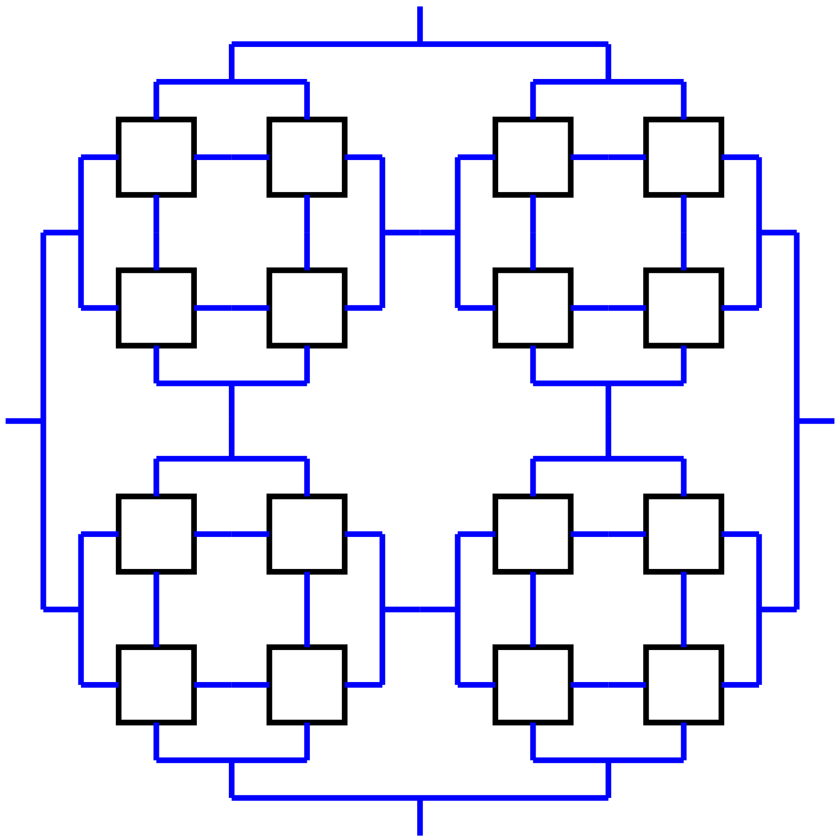
Determined by (generalized) EV-decomposition.

embedding

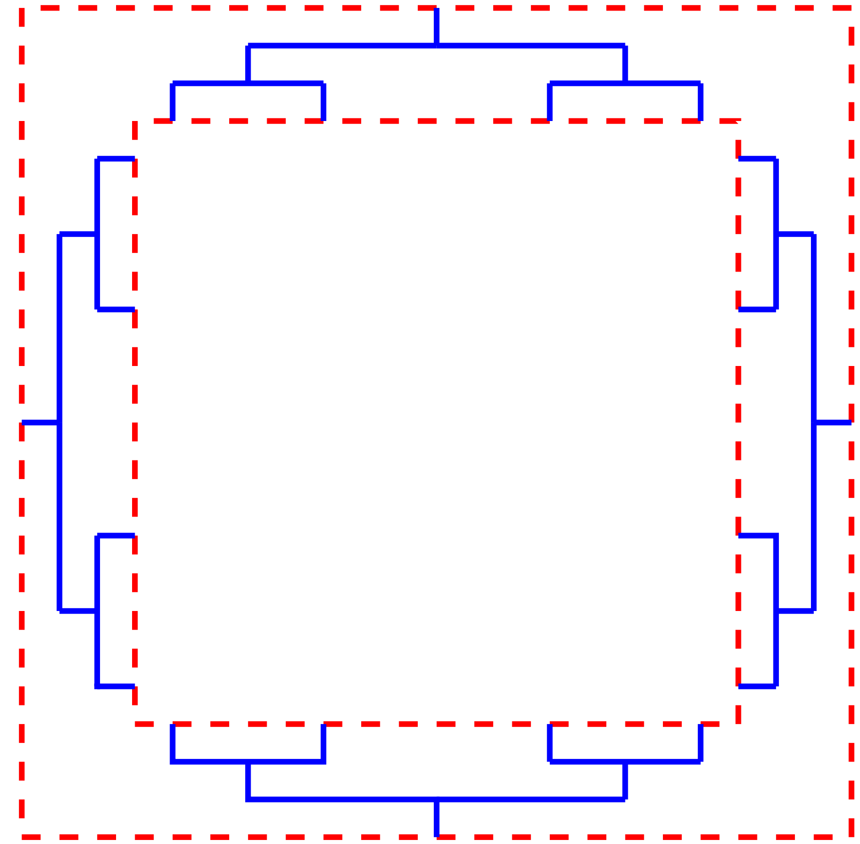
embedding map after 3 iterations



The procedure for 2D state sum

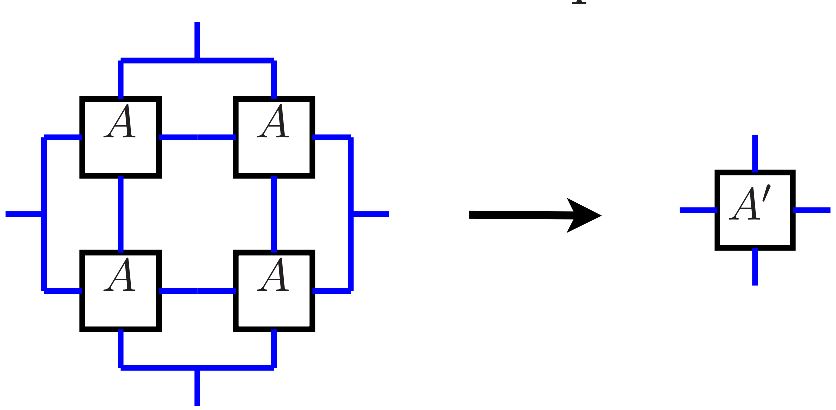


approximation



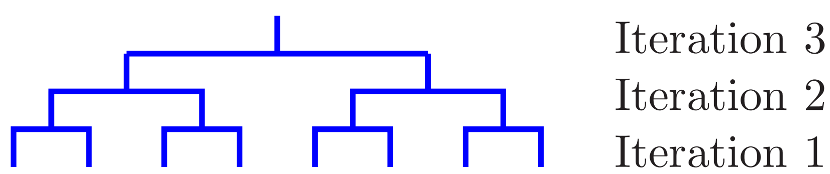
embedding maps
needed to compare results
for different bond dimensions

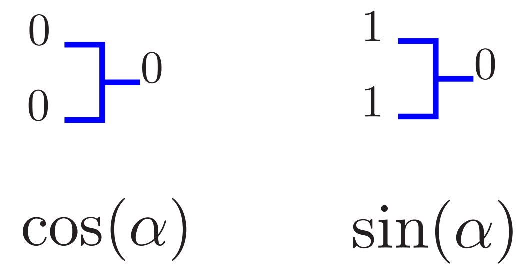
iteration step

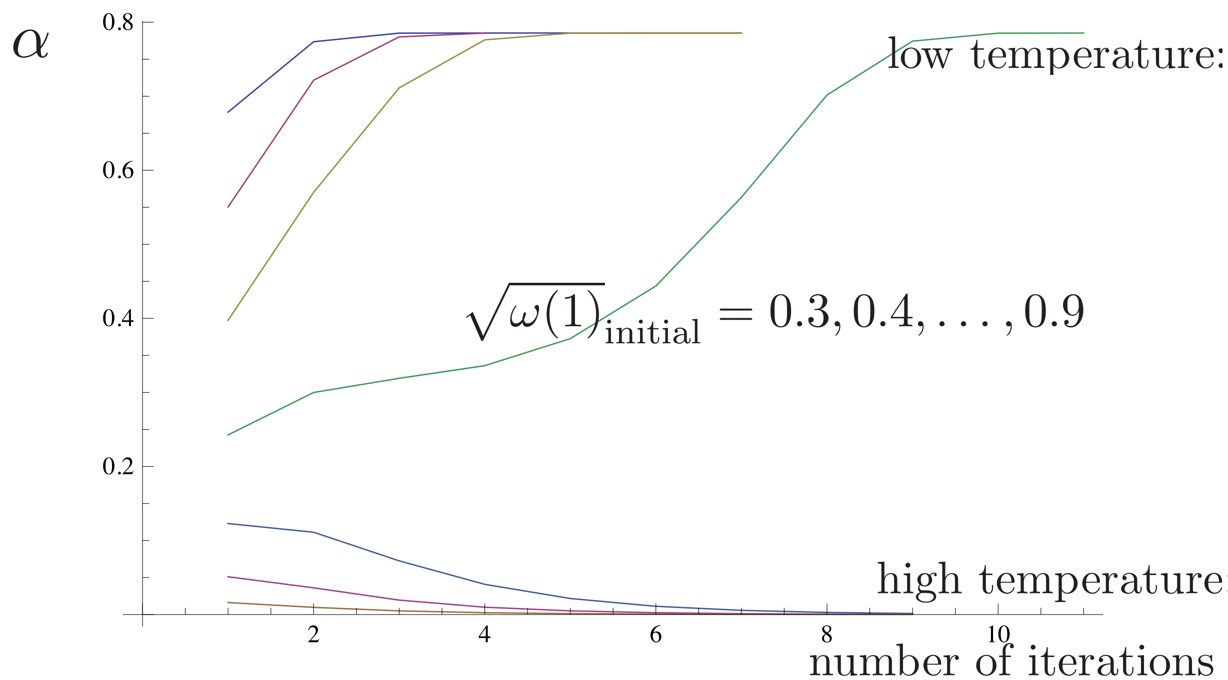


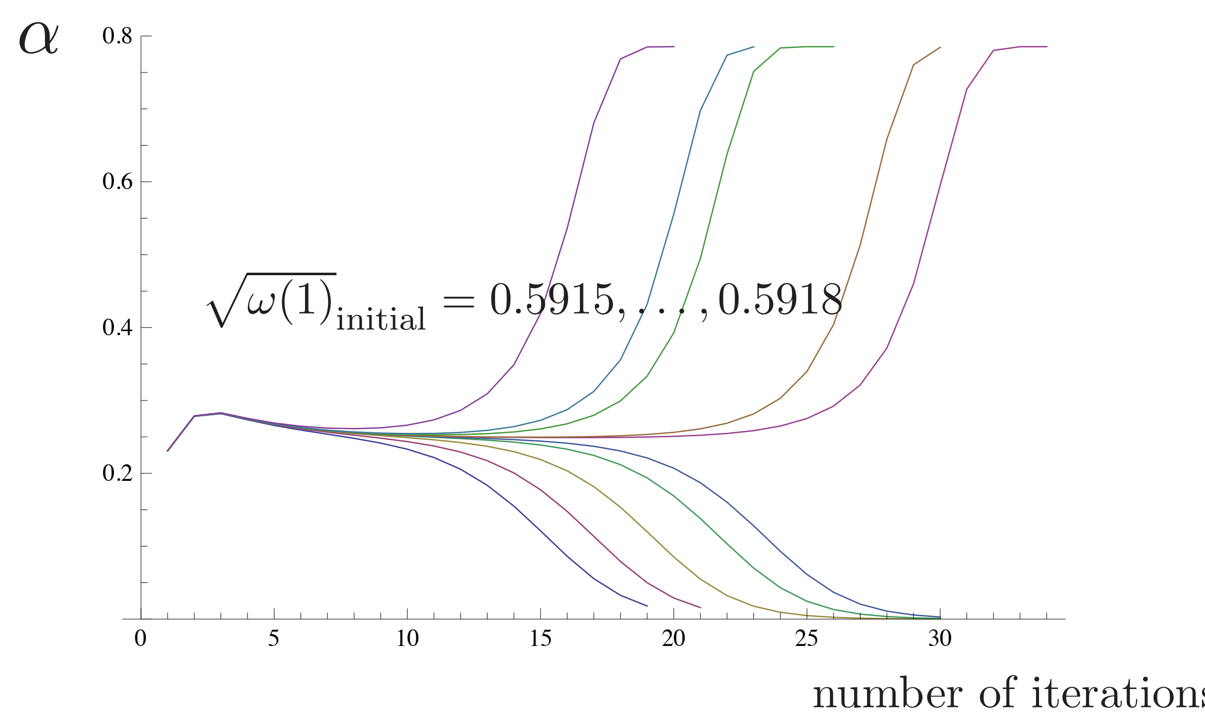
convergence defines continuum limit

Example: Ising model









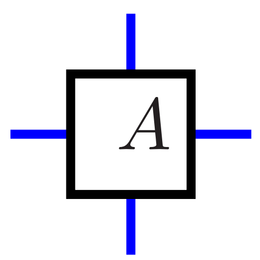
Plateau (scale free dynamics) of almost constant embedding maps around phase transition

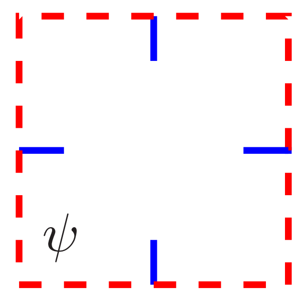
Embeddings determined by the dynamics of the system. Represent the physical vacuum for finer degrees of freedom.

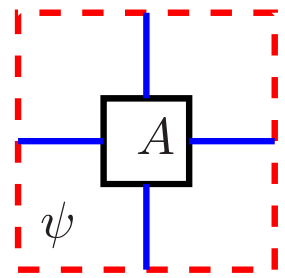
State sums with (generalized) boundaries

State sum models associate amplitudes to space time regions with boundary (data)

[Oeckl 03]







$$A(x_1, x_2, x_3, x_4) = \sum_{x_{\text{bulk}}} a(x_1, x_2, x_3, x_4, x_{\text{bulk}})$$

where x are boundary data

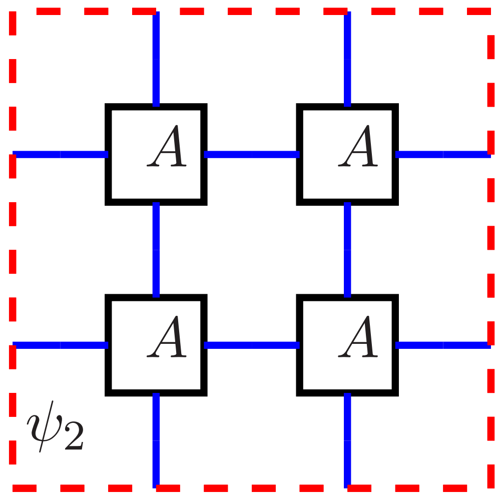
$\psi(x_1, x_2, x_3, x_4)$
is a boundary wave function

A is an (anti-)linear functional on bdry Hilbert space $\mathcal{H}_1$,

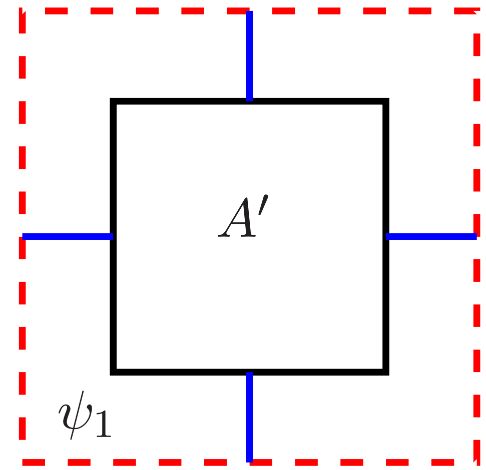
$$A(\psi) = \sum_{x_i} A(x_i)\bar{\psi}(x_i)$$

defines (transition) amplitudes

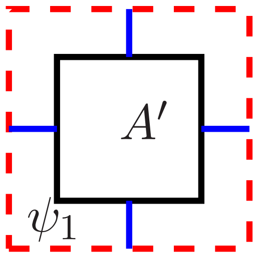
Coarse graining space time regions



Amplitude for a ‘larger’ region glued from amplitudes of smaller regions, acts on ‘refined’ bdry Hilbert space $\mathcal{H}_2$

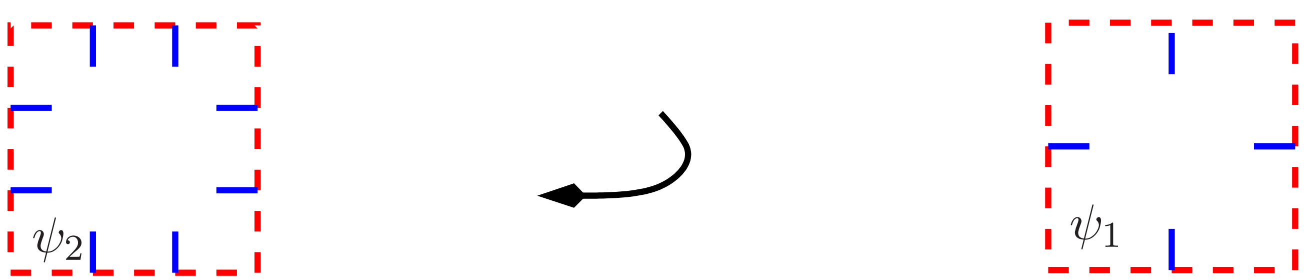


We want to define an effective amplitude acting on coarser boundary Hilbert space $\mathcal{H}_1$



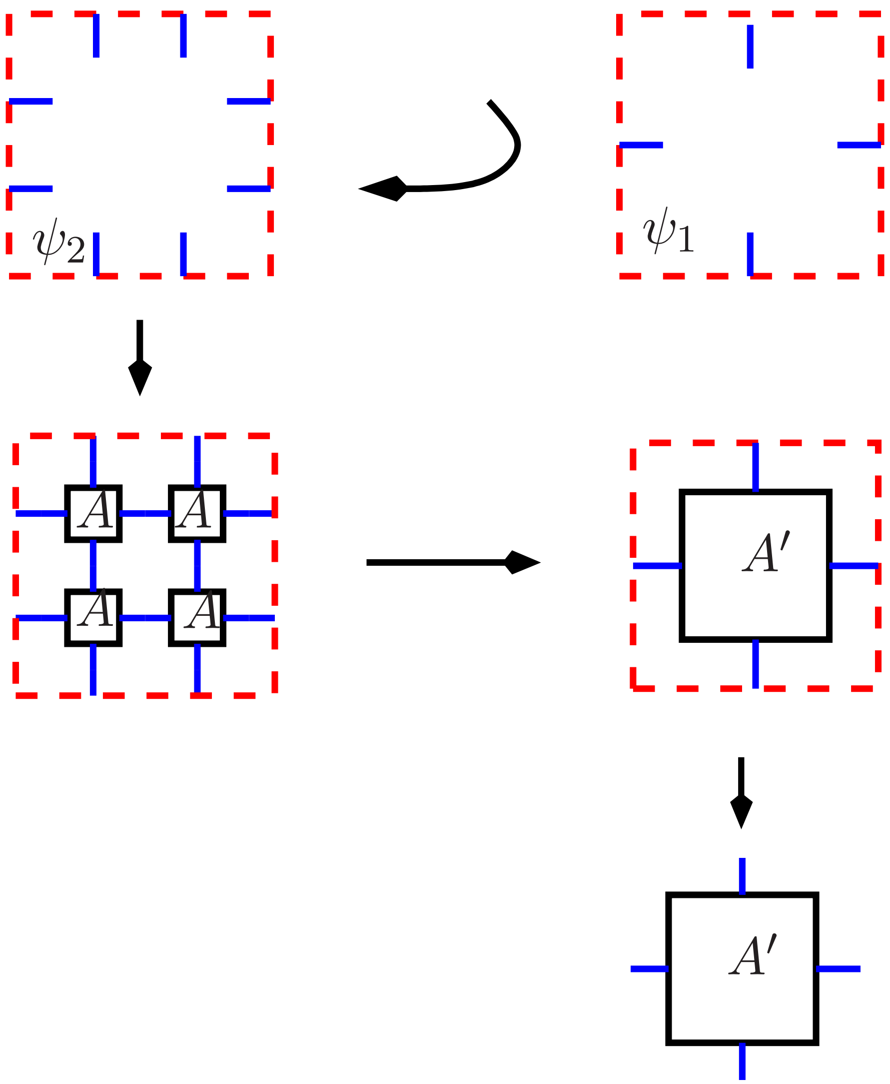
Take (rescaled) effective amplitude as new amplitude for original region

(no rescaling necessary for gravity or reparametrization invariant systems)



Need to relate coarser and finer bdry Hilbert spaces by embedding maps

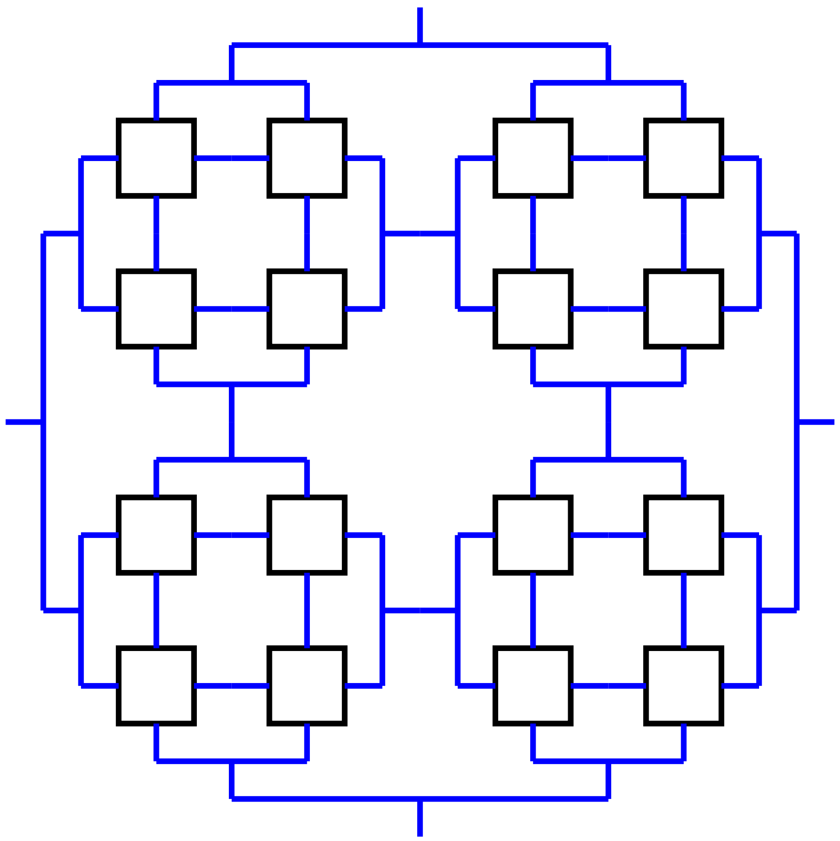
Embedding boundaries



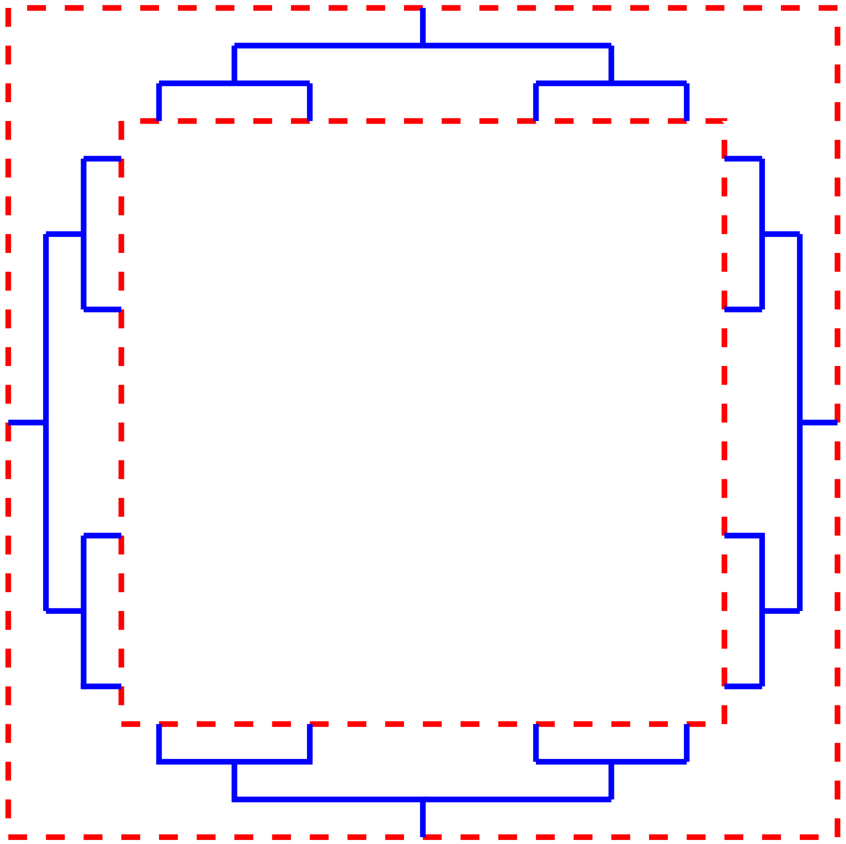
Via the embedding map we can find the effective amplitude functional A' on $\mathcal{H}_1$.

Take A' as new amplitude functional. Iterate and find fixed point.

The procedure for 2D state sum



approximation



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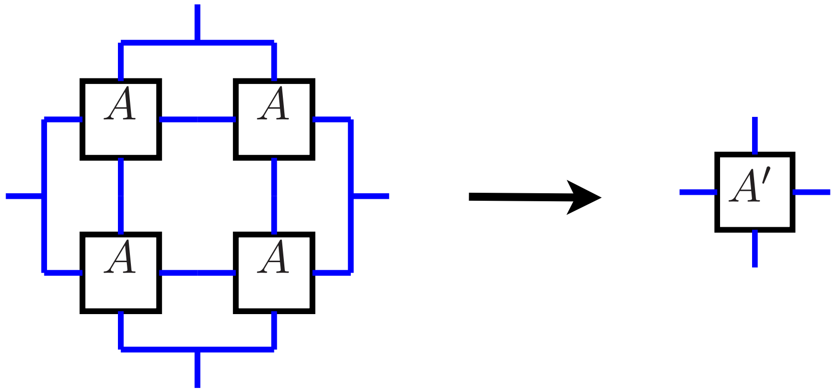

➡Coarse graining transformation.
➡Convergence defines continuum theory.
➡Nested embedding maps define vacuum.

Application to spin foams /nets

[BD,Eckert, Martin-Benito, Steinhaus 2011-13]

- Vacua related to possible phases (infrared fixed points) of the theory
- Is there a phase which lead to smooth space times?

- investigations in (analogue) spin foam models lead so far o encouraging results

  [BD, Eckert, Martin-Benito, Steinhaus 2011-13]

- different fixed points found from which one can extract different systems of embedding maps:

  would correspond to different (non-) realizations of 'emergence of space time'

- outlook: expansion of the theory around these vacua

Outlook

At the very foundations of philosophy and (2500 years later) physics ...

What is (quantum) space time?

We will have an answer this question ... in the near future.