PHILOSOPHICAL PATHS INTO STRING THEORY†

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Disclaimer: the following are notes for a talk (given at UIC on 9/28/13 – see beyondspacetime.net). They are a work in progress, and in particular I want to apologize in advance to anyone whose work is inadequately cited here. However, please do feel free to share this work (including this disclaimer). 10/15/13

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This paper sets out some of the features of string theory that are most salient for philosophers interested in the theory. One goal is to convince philosophers of physics that the formalism is not to be feared – that familiar physical intuitions, and knowledge of general relativity and quantum field theory will carry you a long way. In particular, the first section attempts to give an overview of the formalism of string theory, a kind of large scale road map of what you will find in textbooks; something to help you keep the big picture in mind as you work through the details.[1] Another goal is to pose a series of question that I believe should be addressed by philosophers of physics – my hope is that these will convince the reader that string theory is ripe with philosophical issues, that make an investment in learning the formalism well worth it.

† I DEDICATE THIS WORK TO BOB WEINGARD.

[1] Especially, [Becker et al.(2006)Becker, Becker, and Schwarz, Kiritsis(2011), Polchinski(2003), Zwiebach(2004)].

1. The Formalism

1.1. **The Classical Action.** We will start with a classical relativistic string, an object of one spatial dimension and one temporal dimension – it's best to think of it as a spacetime object from the get go. Let us suppose that it is 'closed', meaning that its ends are joined into a loop (figure 1.1 shows an open string – to close it, the timelike edges should be identified). Our string is free, subject to internal tension, but (for now) under the influence of no external forces, including gravity, so that it lives in Minkowski spacetime, with metric $\eta_{\mu\nu}$. Suppose that the points of the string come labelled with 'internal spacetime coordinates' σ and τ (sometimes σ_0 and σ_1); while 'external' or 'target' spacetime has coordinates X^μ ($\mu = 0, 1, \ldots, d-1$). Then we can describe the string worldsheet in spacetime by assigning appropriate inertial coordinates $(X^0, X^1, \ldots, X^{d-1})$ to each internal point (σ, τ); formally there is d-component vector *field* on the string. From the point of view of the string then, motion in target space amounts to changes in this field. This picture will be important as we progress, so bear it in mind.

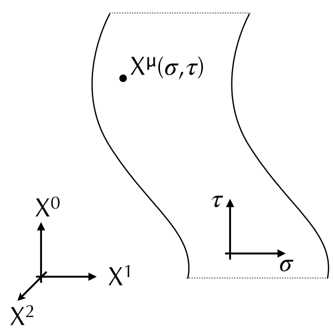

FIGURE 1. An open string in target space – if the timelike edges are identified then it becomes a closed string.

So how do we expect this 2-dimensional object to behave? One's mind might first turn to Hooke's law, but that is uncongenial to relativity – Lorentz contraction should not change the tension in string. What Hooke's law tells us more generally is that a spring will minimize its length: again, not a relativistically invariant statement, but close to one – the relativistic theory statement is that a string will minimize its *spacetime* area. Thus the simplest classical, relativistic string action is proportional to the invariant area $S = -T \int \mathrm{d}A$. Explicitly, $\mathrm{d}A = \sqrt{-g} \cdot \mathrm{d}X^\mu \mathrm{d}X^\nu$, or transforming into string coordinates, we obtain the famous Nabu-Goto action:

$$S_{NG} = -T \int \mathrm{d}\sigma^2 \sqrt{-\det\Big(\eta_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta}\Big)}. \tag{1}$$

T is the tension in the string (though you can't immediately see this from the form of the action); it makes clear that the string does not satisfy Hooke's law, because it is an invariant *constant*. The action also shows that all that matters (in an inertial frame) is the total length of the string, not how parts might be stretched relative to one another – again un-Hooke-like behaviour. So, for one thing, the dynamics has no way of identifying parts of the string over time (the action has diffeomorphism symmetry with respect to the σs) – but the significance of this behaviour is far greater. (Quick aside: in fact one can give a Hooke's law treatment of the string, but not in inertial coordinates, but in 'light cone' coordinates, in which one spatial coordinate is 'boosted to the speed of light'. Such coordinates are used in most text-books at some point.)

The square root in the Nabu-Goto action is a problem for quantizing the string, so one employs a trick to write down an equivalent action (for details see any recent text-book) – the sigma (or Polyakov) action:

$$S_\sigma = -\frac{T}{2}\int \mathrm{d}^2\sigma\sqrt{-h}h^{\alpha\beta}\eta_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta}. \tag{2}$$

The 'trick' involves introducing a second 'internal' metric, $h_{\alpha\beta}$ on the string worldsheet – to be carefully distinguished from the restriction of the spacetime metric to the string. Now the un-Hooke-like behaviour of the string manifests itself in the fact that intervals with respect to the internal metric have no physical significance. The action appears to depend on how the string is stretched along its length – the derivatives are determined by the distance in external space separating points on the worldsheet that are separated by an infinitesimal distance in the string coordinates. But the behavior of the string that we have been stressing means that such infinitesimal distances have no physical significance, and so it should make no difference if any is rescaled by an arbitrary factor. In short, the action must be Weyl, or conformally invariant, and indeed it is: as can be readily checked, the sigma action is unchanged by replacing $h_{\alpha\beta} \to e^{\phi(\tau,\sigma)}h_{\alpha\beta}$.

A couple of short notes. First, the action is conformally invariant with respect to the string metric, not the target space metric! For g the relevant symmetry is Poincaré invariance. (The other symmetry of the action is diffeomorphism invariance with respect to both the σs and Xs.) Second, although I have been stressing the connection of conformal invariance to the un-Hooke-like behavior of a relativistic string, I did so mainly to illustrate how string theory is grounded in some very familiar physics (to reassure philosophers who might be put off by later complexities). Conformal invariance will be an important strand in what follows here, but all I need is the straight-forward mathematical fact that the action has that symmetry – not any story about why. For now we have the following: from the point of view of the string, string theory concerns a d-dimensional conformal field, living on a 2-dimensional spacetime (i.e., the worldsheet). That picture was central to the developments of the 'second string revolution' of the 1990s, and will be the picture I generally want to develop here.

Question 1: What in general is the consequence of conformal symmetry for a physical theory? E.g., free Maxwell field and $N = 4$ supersymmetric Yang-Mills theory. And how does string theory compare to those?

The symmetries can be used to set the worldsheet metric flat, in which case the action becomes:

$$S_\sigma = \frac{T}{2}\int \mathrm{d}^2\sigma \dot{X}^2 - X'^2. \tag{3}$$

Where the derivative are with respect to the worldsheet coordinates.

The corresponding Hamiltonian is:

$$H = \frac{T}{2}\int d\sigma\ \dot{X}^2 + X'^2; \tag{4}$$

while it's straight-forward to confirm that minimizing with respect to X^μ yields a wave equation (and, for an open string, boundary conditions).

$$\ddot{X}^\mu - X''^\mu = 0. \tag{5}$$

The general solution for a closed string is (after a little more work in classical wave physics):

$$X^\mu = X_0^\mu + \ell_s^2 p^\mu \tau + i\frac{\ell_s}{\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}\big(\alpha_n^\mu e^{-i2n(\tau-\sigma)} + \bar{\alpha}_n^\mu e^{-i2n(\tau+\sigma)}\big), \tag{6}$$

where ℓ_S, the 'characteristic string length', is determined by the tension: $\ell_s^2 = 1/T$. This equation describes an initial position, linear momentum, and left- and right-moving vibrations – the α_n are the amplitudes of the modes of the string. Identifying the linear momentum as the zeroth mode of the string will be useful.

$$\alpha_0^\mu \equiv \frac{\ell_s}{2}p^\mu \equiv \bar{\alpha}_0^\mu. \tag{7}$$

Substituting the mode expansion of X^μ into the string Hamiltonian (4) gives

$$H = \sum_{n=-\infty}^{\infty}(\alpha_{-n}\cdot\alpha_n + \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n) = 0. \tag{8}$$

1.2. **Immediate Consequences.** Now, because of conformal symmetry, the variation of the action with respect to variations in the metric must vanish:

$$0 = \frac{1}{\sqrt{-h}}\frac{\delta S_\sigma}{\delta h^{\alpha\beta}} = -2T\pi\Big(\partial_\alpha X\cdot\partial_\beta X + \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1\end{pmatrix}_{\alpha\beta}(\dot{X}^2 - X'^2)\Big). \tag{9}$$

The four equations given by the possible values of α and β can be solved, and if the expansion for X^μ is inserted, entail that:

$$\forall m \in \mathbb{Z} \quad 0 = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{m-n}\cdot\alpha_n \equiv L_m. \tag{10}$$

The L_m and $\tilde{L}_m$ are crucial objects in the formalism describing the string, the '*Virasoro generators*', and the constraints (10) play a vital role in the theory. Physically speaking, from the worldsheet perspective, (9) gives $T_{\alpha\beta}$, the stress-energy tensor of the 2-dimensional stringy spacetime, and the Virasoro generators are its modes. Geometrically speaking, they are the generators of conformal transformations on the worldsheet.

As an example, consider the role of the constraints in determining the mass spectrum of the string. Observed at scales well above its characteristic length, intuitively a string will appear as a (spatially) point-like object – a particle – since its extension 'can't be seen'. Since the string appears as a particle, its linear four-momentum must satisfy the usual relation. Using (7):

$$-M^2 = p^2 = \frac{2(\alpha_0^2 + \tilde{\alpha}_0^2)}{\ell_s^2}. \tag{11}$$

But the $m = 0$ Virasoro constraint yields

$$L_0 = \frac{1}{2}\sum_{n=-\infty}^{\infty} \alpha_{-n}\cdot\alpha_n = 0 \quad \Rightarrow \quad \frac{\alpha_0^2}{2} = -\sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n, \tag{12}$$

and similarly for $\tilde{L}_0$. So

$$-M^2 \;=\; \frac{4}{\ell_s^2}\sum_{n=1}^{\infty}(\alpha_{-n}\cdot\alpha_n + \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n) = \frac{4}{\ell_s^2}H. \tag{13}$$

In other words, it follows from the constraint that the 'particle-mass' of a string depends on its vibrational modes – different vibrations give different 'particles'. We also substituted our earlier expression for the Hamiltonian (8) to show that the Hamiltonian is proportional to the mass squared of an excited string, not (as might have been expected in relativity) the mass.

1.3. **Quantization.** In this talk I will draw on both canonical and path integral quantization: either way, X^μ is a field on a 2-dimensional Minkowski spacetime – the string worldsheet. Thus we start by imposing equal-time commutation relations on the 'field':

$$[X^\mu(\sigma), \Pi^\nu(\sigma')] = i\eta^{\mu\nu}\delta(\sigma - \sigma'). \tag{14}$$

Here $\eta^{\mu\nu}$ is again the Minkowski metric.[5] It follows from the explicit forms of the canonical field and momentum that the modes satisfy the CCRs

$$[\alpha_m^\mu, \alpha_n^\nu] = m\eta^{\mu\nu}\delta_{m+n}, \tag{15}$$

which (up to a scaling factor) mean that the quantized αs are raising and lowering operators, as one should expect. Hence our earlier analysis shows that the quantized mass spectrum is discrete – including massless photons and gravitons. (Now, for suitable string tensions, the modes can reproduce the observed masses of meson families. However, the appropriate tension for quantum gravity is far greater, so observed particles are *not* theorized to be mode excitations of the string. The mass spectrum of the standard model, beyond the photon, is reproduced in a more complex way – relying, for instance, on compactified dimensions or D-branes.)

Imposing the Virasoro constraints requires that the generators be expressed as operators. The CCRs tell us that for all but $m = 0$ this can be done by replacing the *alpha*s with operators; while L_0 requires normal ordering. The resulting commutation relations are (there is a quick and dirty way to get this result, but more a more careful treatment involves Fadev-Popov quantization):

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{d}{12}m(m^2-1)\delta_{m+n,0}, \tag{16}$$

which is the (classical) algebra of conformal generators, plus a 'central charge' term. That in turn indicates a quantum anomaly in the theory, a breakdown of classical conformal invariance. In short, restoring the symmetry requires that $d = 26$ (which also cures another anomaly in the Lorentz symmetry of the theory). Hence, the infamous compactified dimensions of string theory. (Although 'compact' has a topological meaning, which could equally well apply to the large dimensions of experience if they are closed, I will generally use the term to refer to the small dimensions of string theory.)

Question 2: Should compact dimensions be considered spatial in the same sense as the large ones of experience – or are they better thought of as representing internal degrees of freedom? I.e., are there literally 26 spatial dimensions in bosonic string theory?

2. Duality

Consider a closed string in a space with a compact dimension, x, of radius R. The string may or may not enclose the dimension, and if it does, it may be wound any number of times: call this number, w, the 'winding number' (see figure 2). Moreover, if the strings are interacting, the winding number is a dynamical quantity, which may change.

The state of an unwound string is given by:

[5] So note that the $\mu = \nu = 0$ CCR has an unexpected sign, because it comes from the time-time component of η.

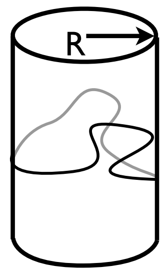

FIGURE 2. A closed string wound once around a closed spatial dimension.

$$X = x_0 + \ell_s^2 p\tau + \text{vibrational terms} \tag{17}$$

p represents the linear momentum of the string, and x_0 the initial position of the string's center of mass. We will focus on the components of the state in the x direction – the x coordinate of the center of mass, the linear motion around x, and (if we were to include them) vibrations in the x direction.

If a string is wound around a dimension, then we can no longer describe it by vibrations about a point – the center of mass – because even without vibrations it is extended. Thus, x_0 must be replaced with a function describing a string wound a whole number w times around x. Suppose there is no motion, then setting the string coordinate, σ, even with the spatial coordinate (and remembering that σ, the spacelike worldsheet coordinate, ranges between 0 and 2π), we find the equation for the string is

$$X = w\sigma R. \tag{18}$$

Any motion – momentum or vibrations – are displacements from this state. Hence, ignoring vibrations

$$X = w\sigma R + \ell_s^2 p\tau. \tag{19}$$

The Hamiltonian is still given by (4):

$$H = \frac{T}{2}\int d\sigma\ \dot{X}^2 + X'^2.$$

Substituting we obtain, for a string wound w times,

$$H = \frac{T}{2}\int d\sigma\ (\ell_s^2 p)^2 + (wR)^2. \tag{20}$$

The next step is to quantize this simple system, which has two dynamical degrees of freedom, momentum and winding.

- Momentum: because the dimension is closed $\Psi_k(x) = e^{ikx}$, where $k = n/R$.

- Winding: the eigenvalues should be $w = 0, 1, 2, \ldots$, so $\Phi_l(y) = e^{ily}$ around a circle with coordinate y, but radius $1/R$, where $l = wR$.

Crucially: since the winding number varies, we cannot treat it as a constant, classical c-number of the system, but as a dynamical *quantum* quantity, described by a wavefunction – just like momentum. The circle on which this $\Phi_l(y)$ wavefunction lives can't be in ordinary space, because then we would expect it to describe a second object which we could expect to find in space. Instead, we have to take it as an 'internal' dimension, associated with each compact dimension of space: Witten calls it 'another "direction" peculiar to string theory' [Witten(1996), 29].

Substitute into the Hamiltonian (20), to obtain the spectrum

$$H = \frac{T}{2}\int d\sigma\ (\ell_s^2 n/R)^2 + (wR)^2. \tag{21}$$

It's easy to see that a string of wave number n and winding number w in a space with radius R has exactly the same value as a string with *winding* number n and *wave* number w, *which lives in a space of radius* ℓ_s^2/R.

$$n \leftrightarrow w \quad \text{and} \quad R \rightarrow \ell_s^2/R \tag{22}$$

$$\Rightarrow \quad (\ell_s^2 n/R)^2 + (wR)^2 \rightarrow \left(\ell_s^2 w \cdot (R/\ell_s^2)\right)^2 + (n \cdot \ell_s^2/R)^2$$

$$= (wR)^2 + (\ell_s^2 n/R)^2. \tag{23}$$

This second string has a spatial wavefunction that lives in a compact dimension of radius ℓ_s^2/R, and hence – by the same reasoning as before – a winding wavefunction that lives in a compact dimension of reciprocal radius, namely R/ℓ_s^2. The Hamiltonian controls the mass spectrum (see ??) and the dynamics, hence: *(i)* because they have the same Hamiltonian, both strings will have the same mass spectrum. Moreover, *(ii)* as can be seen in (23), the roles of momentum and winding are reversed, so that the dynamics of the spatial wavefunction in one string, becomes the dynamics of the winding wavefunction in the other, and *vice versa* – in other words, the correspondence between momentum and winding is preserved over time.

Now, because wavefunctions in physical space are in exact correspondence with the wavefunctions in winding space, every observable pertaining to physical space corresponds to an observable pertaining to winding space – related to winding just as the former is related to momentum. (And *vice versa*.) Then, since momentum and winding are exchanged by (22), every observable pertaining to physical space is exchanged with a corresponding observable pertaining to winding space – and because the wavefunctions are also exchanged, the value of the new observable will equal that of the original observable for the first string. (And *vice versa*.) In short, the pattern of observable quantities will be preserved by (22) – all that changes is whether the quantity is understood to pertain to physical or to winding space.[9]

In other words, we have a translation manual between a pair of theories that agree (under the translation) on the expectation values of all observables in all states, and on the evolutions of all states.

$$\begin{array}{ccc} n & \leftrightarrow & w \\ R & \rightarrow & \ell_s^2/R \\ \Psi(x,t) \rightarrow \Phi(x,t) & \text{and} & \Phi(y,t) \rightarrow \Psi(y,t) \end{array} \tag{24}$$

It's important to realize that 'observables' here does not have a narrow empiricist connotation, but has the formal meaning of the collection of hermitian operators, subject to any (super)selection rules, including gauge symmetries. In other words, the observables are the collection of operators normally thought of as representing the totality of physical, quantum mechanical quantities. And with respect to those quantities the theories are in perfect agreement. This symmetry is known as 'T-duality'.

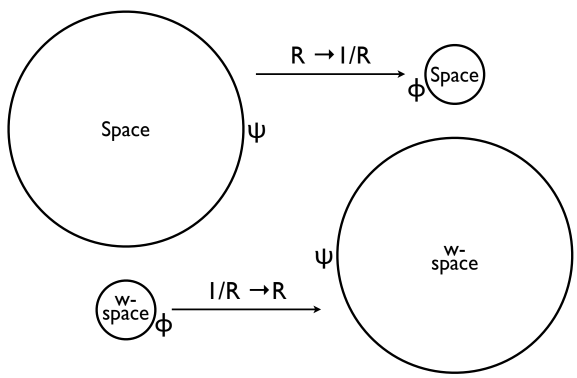

FIGURE 3. T-duality

T-duality implies that any *experiences* of closed dimensions will be of large radii, in the following way ([Brandenberger and Vafa(1989)]). Our experiences depend on low energy processes – the reception of photons, for example. If space is large, $R >> \ell_s$, then spatial wavefunctions have lower momentum, but a wound string will have very high energy, so low energy processes occur in physical space. If space is small, $R << \ell_s$, then the opposite is true – spatial wavefunctions have high momentum, while a wound string will have very low energy, so low energy processes occur in winding space. But in this latter case the winding space is very large, and so dual to a theory with large physical space – and so the

[9]The mapping introduces a ℓ_s^2 factor, but these can be absorbed in a trivial rescaling of observables, so we will ignore it.

low energy processes of experience, although occurring in winding space, will be indistinguishable from experiences of a large space.

Question 3: How should we understand T-duality? Do duals describe physically distinct situations?

Prima facie, in one system the string has momentum n/R, and is wound w times around a dimension of radius R. In the other, it has momentum Rw/ℓ_s^2 and is wound n times around a dimension of radius $R' \equiv \ell_s^2/R$. And in the quantum mechanical treatment spatial and winding-spatial parts of the wavefunction are interchanged (which, in the case of momentum and winding *eigen*states, entails $n \leftrightarrow w$).

Normally one thinks of c-numbers such as R as also physical, in which case the duals describe different physical situations. But normally, c-number parameters can be determined by the values of quantum quantities: the charge on the electron, say, by scattering probabilities. A duality means they cannot be so determined by the values of the observables of the theory: the pattern of expectation values is preserved. So we should at least leave open that these differences in the c-numbers do not, after all, represent physical differences.

- First interpretive decision: either the T-duals agree on the physical world, or they do not. If they do ...
- Second interpretive decision: do both say that strings literally live in a space of radius R (but represent that fact differently); or do they represent facts the same way, and so say nothing beyond their shared consequences – so that the radius is indeterminate between R and $1/R$.

Deciding that the duals do say the same does not settle the issue of what it is that they do say. And here there are two possibilities (or perhaps a range of cases of which these are the extremes). Consider a theory, T, that sets R equal to the observed radius of space

The first interpretive possibility is that T simply says that target space simply is phenomenal space. Then if its dual says the same thing, it must say it in a different way: under the duality, target space is relabeled 'winding space', which now represents phenomenal space. Given the different reading, there is no conflict, phenomenal space has radius R.

A second possibility is that the duality does not amount to a relabeling, but rather conflict is avoided because all either dual says is limited to their mutual consequences. Thus neither asserts a determinate radius for target space. In other words, our representations of string theory contain surplus structure – a target space radius – which does not directly correspond to any physical property. One might then desire a deeper theory that quotients out the difference, to reveal the 'true' degrees of freedom, none of which are surplus.

This possibility implies that phenomenal is space is not identical with target space, because the dimensions of phenomenal space have determinate radii, while those of string space do not. How this can be we saw above – our experiences of space will be of a large radius space according to either dual, because our experiences rely on low energy processes, so the large experienced radius is something on which the duals agree. But if phenomenal space is not target space, then it must be derived – or 'emergent' – from string theory.

Moreover, it follows that we cannot think naively of strings as spatial objects, since there is no fact of the matter (even in a quantum mechanical sense) of how many times they wrap around a dimension.

Question 4: Which of these interpretations is correct?

I don't have time to argue for the claim now, but I think it is the indeterminacy view; and those who have written on this topic seem uniformly to agree. Chronologically:

- "The invariant notions of general relativity such as distance may not be invariant notions for string theory on short distance scales." [Brandenberger and Vafa(1989), 393]
- ". . . in string theory one does not really have a classical spacetime, but only the corresponding two-dimensional field theory; two apparently different spacetimes X and Y might correspond to equivalent two-dimensional field theories." [Witten(1996), 29]
- ". . . there is no distinction, no way to differentiate, between radii that are inversely related to one another." [Greene(1999), 247]

To this list let me also add Dean Rickles, who has explored various dualities (cite). My understanding of his view is that he is sympathetic to inferring indeterminacy from dualities, though his aim is, like mine here, to point out the question.

Question 5: Do other dualities have the same interpretational forks? If so, do the same considerations apply? And do they lead us in the same direction?[13]

3. General Relativity from String Theory

Consider the sigma-action, without the assumption that the target spacetime is Minkowski, but rather has an unspecified metric:

$$S_\sigma = -\frac{1}{\alpha'} \int \mathrm{d}^2\sigma \, \sqrt{-h} h^{\alpha\beta} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu. \tag{25}$$

α' is (up to a factor) the reciprocal of the tension – in worldsheet perturbation theory, an expansion parameter. Otherwise, all we have done is replace the Minkowski dot-product between the X^μ with a general semi-Riemannian metric, $G_{\mu\nu}$. At this point one might feel that G is free parameter in the theory, to be inserted by hand – that the 'background' metric is independent of what the strings do. But you would be wrong – G has to satisfy the source-free Einstein field equations (a result going back to [Friedan"(1980)]; and conjectures going back to the 1970s). Yes, string theory – especially its conformal symmetry – requires general relativity (to lowest order).

[13] See especially [Teh(2013), Rickles(2011), Rickles(2012), Rickles(2013)].

The proof runs this way ([Callan et al.(1985)Callan, Friedan, Martinec, and Perry], Gasparini for detail) – Tiziana will elaborate (for even more detail, see her Santa Cruz talk on YouTube):

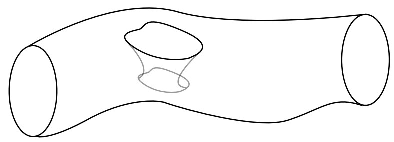

FIGURE 4. A string worldsheet with a 'hole' through it. This surface represents the first order correction to the closed string propagator – time is running left-to-right.

(1) Consider the first order correction in string perturbation theory – topologically, a surface with one hole, a torus with legs (see figure).
(2) The contribution of this term comes from a QFT, X^μ, on this 2-D worldsheet, defined by the action –a 'non-linear sigma model', in which $G_{\mu\nu}$ acts like a coupling (one whose value varies across the worldsheet).
(3) In turn, this theory has a well-studied perturbation theory, and is known to be renormalizable. Note that there is a second question of the renormalizability of the string perturbation theory, the series of surfaces.
(4) Renormalization means counter-terms, which means running terms, with a length dependence. The renomalization group studies this dependence, in particular describing the behavior in terms of a $\beta-$function – generally computed perturbatively.
(5) Friedan studied the renormalization group behavior of this theory, and showed that in the sigma model perturbation theory, the β-function associated with $G_{\mu\nu}$ is given by $\beta_G = R_{\mu\nu} + O(\alpha')$, the Ricci tensor plus higher order corrections.
(6) Now, recall that we are talking about a field theory on a string worldsheet, so the length scale is with respect to the string metric, rather than the target space metric – but conformal invariance means that the theory can have no dependence on such a length! So the β-function vanishes, and by Friedan's work, $R_{\mu\nu} = 0$, or $G_{\mu\nu}$ is 'Ricci flat' to lowest order. But that's equivalent to the vacuum Einstein field equations.

Of course, spacetime is not Ricci flat, so the question arises of whether other solutions are possible. And they are, for the result generalizes. To take a particularly salient example, suppose that strings live in a target space with a Yang-Mills field – imagine a string theory with familiar matter. Such a system has an action of the form:

$$S_\sigma \sim \frac{1}{\alpha'} \int \mathrm{d}^2\sigma \mathrm{d}\theta \; G_{\mu\nu}\partial X^\mu \partial X^\nu + A_{\mu a}(X)\partial X^\mu j^a + \theta\psi^i\partial\psi^i. \tag{26}$$

This action describes a 'heterotic' string, not the bosonic string we have considered so far. Such a theory has both conformal symmetry and supersymmetry between bose and fermi degrees of freedom – θ represents fermionic 'coordinates' in addition to the bosonic' σ. The

A-field is the background gauge field, and ψ the fermions to which it couples (and j their current). Callan et al investigated this action, in the same way, showing that to lowest order once again $G_{\mu\nu}$ is Ricci flat, and that the standard free Yang-Mills equations must be satisfied. That doesn't reassure us that string theory is compatible with a universe like ours which is not a vacuum, but happily they also showed that at first order in α' the β-function for the target metric has a term corresponding to the stress-energy of the Yang-Mills field. Thus, once again, worldsheet conformal symmetry means that to order α', the Einstein field equations hold.

$$\beta_G = R_{\mu\nu} - \frac{\alpha'}{2}\alpha' \mathrm{tr}(F^2_{\mu\nu}) + O(\alpha') \text{ terms for the coupling to } G_{\mu\nu}. \tag{27}$$

Thus, string theory formally implies general relativity.

Question 6: What is significance of these results?

A possible answer (to which I am very sympathetic, and seems to be a popular view among string theorists): 'background' fields do not represent new degrees of freedom in addition to those of string theory – speaking in an ontological mode, they are not distinct primitive entities. Instead they represent the behavior of coherent states of string excitations: the quantum states, that is, which describe classical field behavior. Thus when one includes a general metric in the action, one has a quantum theory of perturbations around a coherent state corresponding to the given classical metric – quantum corrections to (almost) any classical spacetime. The result shows that there is not a free choice of background fields, but that graviton and – in this case – Yang-Mills quanta coherent states must be related appropriately: by the EFEs.

More precisely, there are two claims involved in the view that background fields represent coherent states (both with evidence in their favor): (i) that the string is an adequate TOE, in the sense that the string spectrum includes quanta for all the background fields desired; and (ii) that the terms in the action accurately capture the effective behavior of those coherent states. Then by (i) the $G_{\mu\nu}$ field is composed of stringy excitations, and by (ii) it satisfies the Einstein Field Equations – making it the gravitational field, and the excitations gravitons. In other words, in the most literal sense the general relativistic theory of spacetime is a low energy effective theory of strings.

If this is correct, then it shows that in a certain, rather central sense, string theory is background independent – the curved metric indeed arises from string interactions, rather than being stipulated a priori. Just like general relativity, many solutions are possible, but matter and gravity have to satisfy a mutual dynamics – except in string theory, there is no fundamental distinction between the two (a significant kind of ontological unification). What is left of the charge of background dependence is that the 'background field free' theory, of which the action discussed represents a sector of coherent states, itself involves a metric – gravitons, even in macroscopic coherent states, live in some underlying geometry. It's worth noting then that this metric is, for instance, the Minkowski metric (or something similarly regular). moreover, from the point of view of conformal field theory on the string worldsheet, nothing but an inner product on X^μ fields. In that view, the really interesting geometry of the theory is indeed dynamical, and just an aspect of the same processes that constitute matter.

Question 7: How does the derivation fit within existing accounts of reduction? And from that point of view, does the derivation of the field equation amount to a demonstration that 'spacetime' emerges?

Question 8: The results as stated apply to a target space formed from large and compact dimensions. In physically realistic situations the low order approximation should hold for the large dimensions, but does it hold for the compact ones?

I suspect that there are known answers to the question of the applicability of the results. If the answer is in the negative, then what I see is the possibility of an argument that the compact dimensions are not spatial in the familiar sense, but internal degrees of freedom – that there is an important sense in which space is 3-dimensional after all!

4. Black Holes

5. D-branes

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