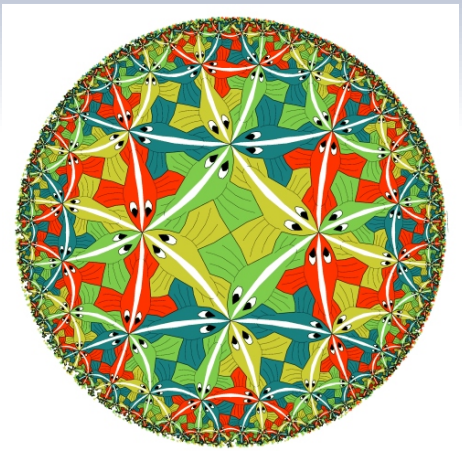


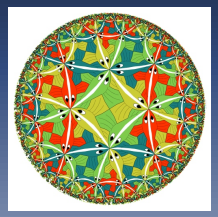
The Dimensions of Duality

Eleanor Knox

Seminar on the Philosophical Foundations of Quantum Gravity, Chicago

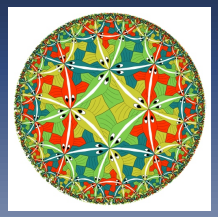
28.09.13

King's College London



Aims

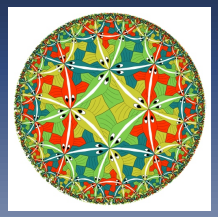
To suggest an approach to thinking about cases in which we have two descriptions of different dimensionality that are 'equivalent'. Clear thinking about a more familiar case (gauge theories on fibre bundles) can help when we come to think about string-theoretic dualities that involve descriptions of different dimension.



Preview

Holographic principles and conjectures assert an equivalence between higher-dimensional 'bulk' gravitational theories and lower dimensional field theories on the boundary of the higher dimensional space. AdS/CFT is a generic name for a class of such conjectures related to string theory. Such "dualities of dimension" raise philosophical issues:

- Should we see the two descriptions as representing the same reality?
- If so, how can we make sense of representations with different dimensionality? Doesn't our fundamental picture have to have determinate dimensionality?

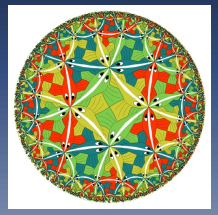


Preview

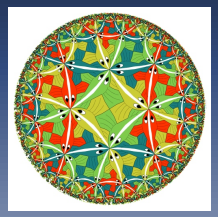
But this is not the first time we've come across formulations of theories with different dimensionality! Two examples:

- Gauge theories: geometrical vs ordinary spacetime representations of gauge theories - "fibre bundle substantivalism" vs holonomies approach.
- (Quantum mechanics: representing the quantum state on configuration space vs ordinary space - "wavefunction/ configuration space realism" and "spacetime state realism".)

Consideration of what these debates are about will shed some light on the AdS/CFT case. I'll argue for an approach on which questions about spacetime are divorced from questions of fundamentality.



Outline

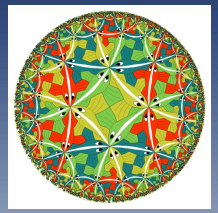


Dimension in gauge theories

What's the basic ontology of a gauge theory like classical electromagnetism? Two (of many) options corresponding to different representations of electromagnetism:

1. **Holonomies approach**: (Healey, Belot) Take the holonomy on a curve $h(\gamma) = e^{-\frac{iq}{\hbar}\oint \mathbf{A}(\mathbf{r}).d\mathbf{r}}$ to represent the basic electromagnetic properties of paths in ordinary spacetime.
2. **Fibre bundle substantivalism**: (Arntzenius, hinted at by Batterman, Nounou) Note that we can represent via a U(1) fibre bundle over spacetime with a connection corresponding to the electromagnetic potential. Reify this higher-dimensional space:

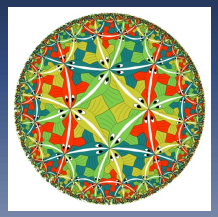
> *...one should take fibre bundles (and their parts) to be the objects that exist...Furthermore, we should take the connections and the sections of those fibre bundles to correspond to fundamental properties of the fibre bundle... [Arntzenius, 2012]*



Dimension in the AdS/CFT correspondence

One version of the AdS/CFT correspondence asserts the equivalence of two representations/theories (in the rather deep sense that these have isomorphic Hilbert spaces - beyond empirical equivalence):

1. 10-dimensional Type IIB string theory on $AdS_5 \times S_5$ space. (5 dimensional anti-de-Sitter space with 5 compactified dimensions).
2. A four-dimensional conformal field theory on the boundary of that space.

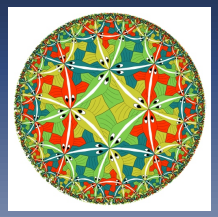


Interpreting AdS/CFT

If we accept the equivalence, we then have three realist interpretational options. (Nb. no-one thinks this is a realistic theory, so this is a hypothetical exercise!)

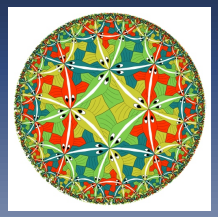
- Insist that the bulk theory most faithfully represents reality, and that the boundary theory is auxiliary to it.
- Insist that the boundary theory most faithfully represents reality, and that the bulk theory is auxiliary to it.
- Hold both theories to represent the same reality.

How are we to decide, and how can we make sense of the third option?



What are these debates about?

There is an arena in which the dynamics does its work, a stage on which whatever theory we happen to be entertaining depicts the world as unfolding, a space (that is) in which a specification of the local conditions at every address at some particular time amounts to a complete specification of the physical situation of the world.



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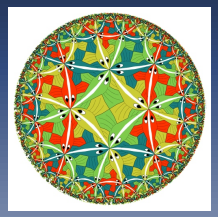
According to David Albert, it's the nature of this arena that's at issue in these debates.



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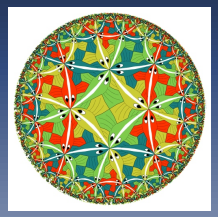

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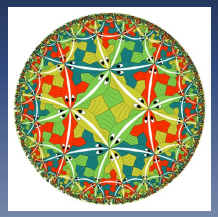

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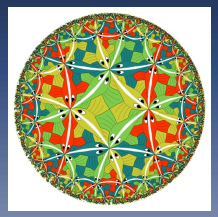

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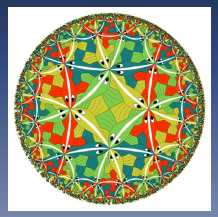

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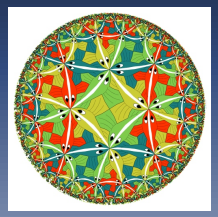
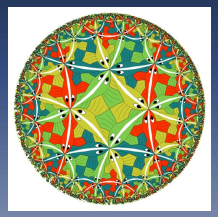
2. Which objects in the theory represent spacetime structure/which degrees of freedom are spatiotemporal?
   - A claim: Spacetime is a functional concept. This question will be answered by seeing which objects play certain kinds of role in our theory.



Spacetime as a functional concept

What does it mean to say that our concept of spacetime is a functional one?

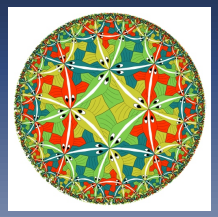
- Contrast membership of a 'functional kind' with membership of a 'compositional kind', where membership/inclusion under the concept is determined by essence.
- Commits us to analysing the spacetime role and identifying what instantiates it (if anything!) in a given theory.
- There's a long and illustrious history of asserting that a concept is functional without filling in the details. (Mental states, genes...)
- But we can at least give a sketch...



A role for spacetime

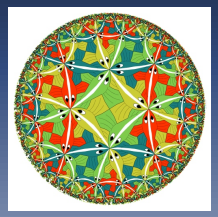
- Most philosophers are committed to the idea that the symmetries of spacetime should match the dynamical symmetries of the theory.
- If we include 'giving the symmetries of the dynamics' in our functional definition, then we'll elevate this from a heuristic to a conceptual truth.
- But the application of this prescription, although still important, is not quite so obvious in e.g. general relativity (where symmetries are generally local, not global).
- And what about other considerations - e.g. that spacetime geometry should govern (reflect?) the behaviour of rigid bodies and periodic processes (where such exist)?

A Claim: The above roles will be filled by an object if it determines a class of (locally) preferred inertial coordinates. (See Harvey Brown's dynamical relativity).



Back to what's at issue

So consider Albert's question of the stage or arena again. If we know which formulation or theory is more fundamental, and we know which objects and degrees of freedom are spatiotemporal, have we answered it? What more can we ask for?



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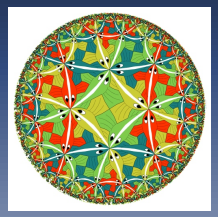

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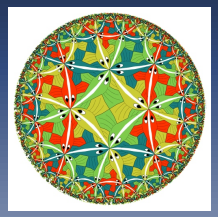

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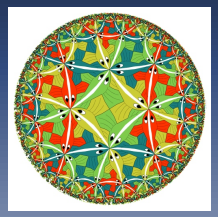

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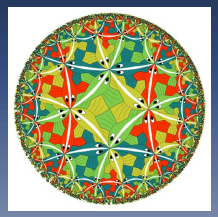

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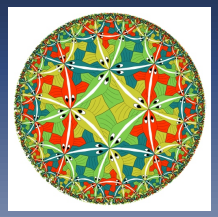
- Q: Where does the stuff in the universe live? What's the container like?
- A: You're pushing metaphors too far into unfamiliar domains.

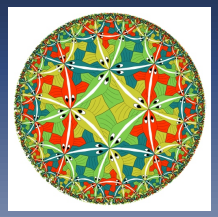
So let's briefly consider how our dimensionality debates answer our questions about fundamentality and spacetime structure...



Spacetime and the fibre bundle

Which version of electromagnetism correctly describes spacetime?

- Both do! Both descriptions include ordinary Minkowski spacetime.
- No argument that the additional space of the fibre bundle fills a spatiotemporal role. (Note that the fibre bundle isn't even invariant under the gauge symmetry, so we can't think of it as encoding additional spacetime symmetries.)

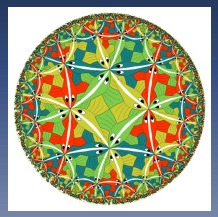


Fundamentality and the fibre bundle

Which version of electromagnetism offers a more fundamental picture?

- The fibre-bundle is not gauge invariant, but we can insist that gauge related fibre bundles represent the same physical reality.
- Once we've done this, the gauge-related classes of bundles represent correspond neatly to the holonomies.
- No considerations of generality to fuel fundamentality claims.
- See both descriptions as representing the same (non-spatiotemporal!) degrees of freedom.

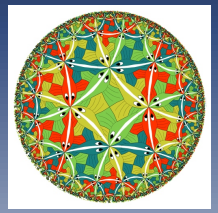
So in light of this, what should we say about AdS/CFT? I'll present some tentative thoughts...



Spacetime and AdS/CFT

Does the bulk or the boundary theory represent spacetime accurately and hence give spacetime dimensionality?

- The functional approach leaves it entirely open that both theories might contain something deserving of the name spacetime - as long as the space in question plays the right role with respect to the rest of the theory.
- The target metric in string theory (or at least its non-compactified dimensions) plays a pretty traditional role in terms of defining inertial frames and therefore seems spatiotemporal. (See NH's talk).
- Inasmuch as it's a field theory with a flat Minkowski metric, seems as if the metric in the boundary theory also plays a fairly standard role. But we do need to bear in mind that we're talking about a conformal field theory...

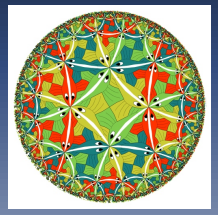


Conformal field theory: some naive comments

(Naive because no mention of supersymmetry)

- Conformal field theories are invariant under conformal transformations of the metric $g_{\mu\nu} \rightarrow \Omega(x) g_{\mu\nu}$.
- This means that (if we're obeying the edict that our spacetime structure should match our symmetry group), the potential representor of spacetime is the equivalence class of metrics conformally equivalent to the Minkowski metric, which would represent a much less structured spacetime.
- It's not obvious that a spacetime with just conformal structure is worthy of the name (nothing like inertial structure here)!
- Moreover (as plenty of authors are aware), this means that the structure is not a candidate for representing phenomenological spacetime; macro-laws are manifestly not conformally invariant!

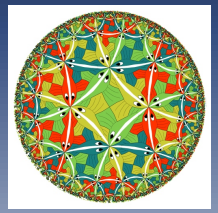
But, of course, not all instances of the holographic principle involve CFTs...



Fundamentality and AdS/CFT

Should we think of either the bulk or boundary theory as more fundamental?

- If we think of the equivalence as precise, it is hard to ground claims of greater applicability or domain. But because the correspondence involves limiting procedures some claim that one emerges from the other.
- It might also be the case that one is more revealing with respect to the structure of some underlying theory...
- But note that there isn't usually a debate over fundamentality in cases of duality - part of the duality lore is that the representations are equally fundamental.
- So one moral of this talk is that differences in dimensionality aren't themselves reason to think there must be a meaningful debate over fundamentality.



Suggestions for philosophers considering dualities of dimension

- Bear in mind that deciding which theory is more fundamental need not depend on which better describes spacetime.
- On the approach here one can ask about dimensionality of spacetime, but questions about the dimensionality of more abstract spaces just boil down to questions about total degrees of freedom.(Note that one can think of the boundary theory as 'internalizing' degrees of freedom that are spatiotemporal in the bulk theory.)
- The approach here pushes us towards accepting multiple representations in non-stringy cases where some philosophers have thought it is important to choose. So perhaps the apparent problems of dual descriptions are less daunting than they appear.

Thank you!