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Making Time for Quantum Gravity

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The Road Ahead

1. A Pre-History of the Problem of Time
   - Heisenberg's Uncertainty Principle
   - Frozen Time
   - Schödinger's Rescue
2. Clocks lead to Event Times
   - Relational Clocks
   - Conditional Probabilities
   - Event Times
3. Events and Histories
   - QM is About Events
   - Histories are About Events
   - Isham's HPO Formalism
4. The Problem of Time
   - Canonical Quantization
   - Auxiliary Histories
   - Relational Quantum Time

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Act I: Quantum Jumps Heisenberg and Jordan interpret the time-energy uncertainty relation in terms of the time of a 'quantum jump.'

Act II: Frozen Time Heisenberg and Born confront the Problem of Time at the universal level.

Act III: Parameter Time Schrödinger unfreezes the formalism by allowing energy superpositions.

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- Heisenberg (1926) analyzes the exchange of energy between two subsystems in terms of quantum jumps.
- Leads to Dirac's Transformation Theory.
- Transformation Theory given physical interpretation by Heisenberg (1927).

Heisenberg's Time-Energy Uncertainty Principle

According to the physical interpretation of quantum theory aimed at here, ***the times of transitions or "quantum jumps" must be as concrete and determinable by measurement as, say, energies in stationary states****. The spread within which such an instant is specifiable is [...] $h/\Delta E$, if ΔE designates the change in energy in a quantum jump.*
(Heisenberg, 1927)

Jordan on the Time of a Quantum Jump

What predictions can our theory make on this point? The most obvious answer is that the theory only gives averages, and can tell us, on the average, how many quantum jumps will occur in any interval of time. Thus, we must conclude, ***the theory gives the probability that a jump will occur at a given moment****; and thus, so we might be led to conclude, the exact moment is indeterminate, and all we have is a probability for the jump. (Jordan, 1927)*

Heisenberg and Born Against Times

At Solvay 1927, this idea was abandoned:

> *If one asks the question* ***when*** *a quantum jump occurs,* ***the theory provides no answer****. At first it seemed that there was a gap here that might be filled with further probing. But soon it became apparent that this is not so, rather, that it is a failure of principle, which is deeply anchored in the nature of the possibility of physical knowledge. One sees that quantum mechanics yields mean values correctly, but* ***cannot predict the occurrence of an individual event****.*
> *(Heisenberg and Born, 1927)*

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Following Campbell (1926):

> *matrix mechanics deals* ***only with closed periodic systems****, and in these* ***there are no changes****. In order to have true processes . . . one must restrict one's attention to a part of the system.*
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> *Limiting our attention to an isolated system,* ***we would not perceive the passage of time*** *in it any more than we can notice its possible progress in space. [In a subsystem w]hat we would notice would be* ***merely a sequence of discontinuous transitions****, so to speak a cinematic image, but without the possibility of comparing the time intervals between transitions.*
> *(Schrödinger, 1927)*

Solving the Problem of Time

In the quantum jump interpretation of matrix mechanics, an isolated system is in an **energy eigenstate**

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This talk:

Consideration of Relational Clocks leads to Relational Time.

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- What event is being assigned probability one?
- How do we assign probabilities to further events?

Conditional Probabilities

- Conditional probabilities are given by Lüders' Rule:

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- 'Given that an event occurs at some time, when will it occur?'
- (e.g.) Decay time of a radioactive atom: atom decays only once so the outcomes 'decay at time t' are exclusive.

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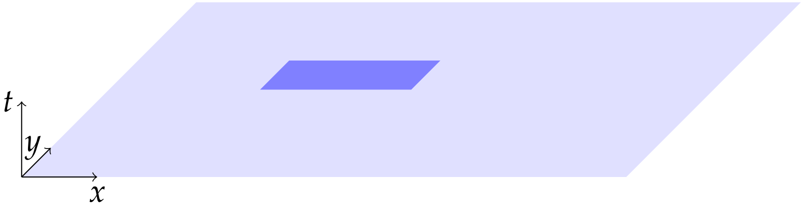

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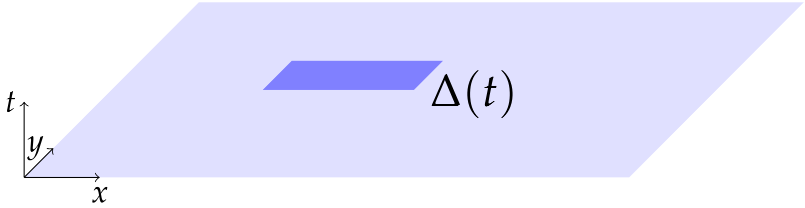

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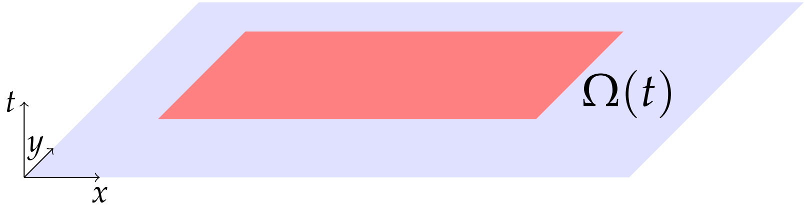

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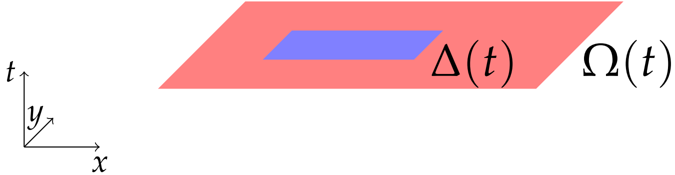

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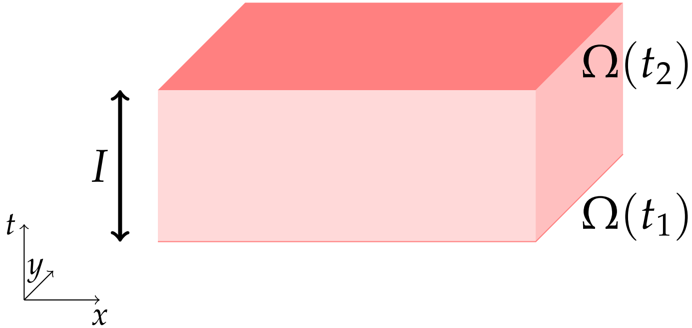

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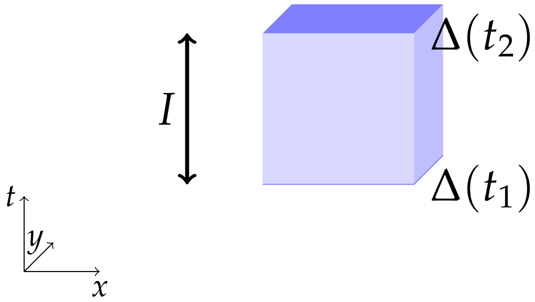

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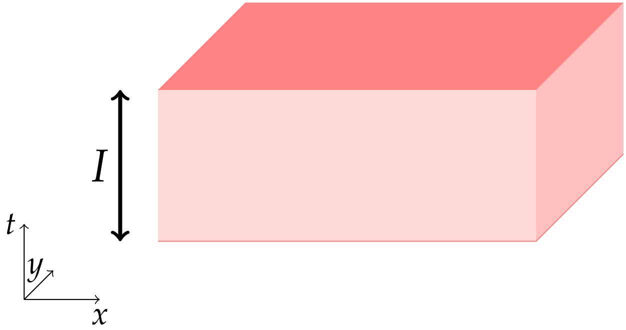

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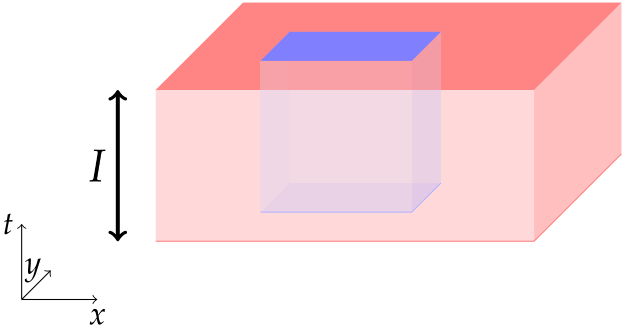

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- NB '$<$' need not even be a partial order on the set of times (can fail to be transitive).

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A history contains a series of propositions about the world. They are true just in case the events they describe **happen**.

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$$d(\alpha,\beta)_{\rho_{t_0}} := \mathrm{tr}\left[C_\alpha^\dagger \rho_{t_0} C_\beta\right].$$

- A family of histories $\alpha, \beta \in F$ where $d(\alpha,\beta) = 0$ for any $\varphi_k \neq \varphi_k'$ is a consistent set.

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- Inspired by decoherent/consistent histories, but more general.
- Orthogonality is guaranteed so no need for decoherence as dynamical process.
- No need for a time parameter: the ordering of temporal propositions is fundamental (see also Wootters, 1994)

Decoherent Histories, Abstractly

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- Abstractly, time is defined as an order on (partial) histories.

Canonical Quantization and Time

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The alternative is to remain in the 'auxiliary' Hilbert space.

The Extended Schrödinger Equation

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This takes place in the so-called **auxiliary** Hilbert space $\mathscr{H}^+$.

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A weight $B \mapsto w(B)$ is an *unbounded* positive linear functional, i.e. $w(B) \in \mathbb{R}_+ \cup \{\infty\}$.

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The weight provides an assignment of expectation values under the condition **that the event does in fact occur**.

$$\omega_\psi(A|B) := \frac{w_\psi\left[B^{1/2}AB^{1/2}\right]}{w_\psi[B]}.$$

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- Conditional probabilities concern successive (partial) histories.

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- Might hope to recover relativistic spacetime as partial order on (highly probable) events.

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- The time translation group for histories is not U_t!

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- Conditional probabilities for successive histories may give a relationist account of time in quantum gravity

Thank you.