First-Class Constraints, Gauge, and the Wheeler–DeWitt Equation

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Classical and Quantum Problems of Time

That the (canonical) quantization of general relativity leads to a timeless formalism should be understood as a consequence of an incorrect treatment of the temporal symmetries of the classical theory. By treating local temporal labellings as entirely unphysical, and change as entirely relational, we do not retain in the quantum formalism the full classical dynamics or the implicit temporal-ordering structure.

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- “Orthodoxy” *is* wrong, but
- Does canonical quantisation rely on it?
- Is the Wheeler–DeWitt equation fundamentally timeless?

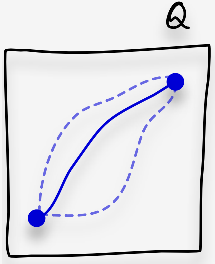
Plan

1. Review of Orthodoxy
2. Challenges to Orthodoxy
    - Pitts
    - Barbour and Foster
3. Time and the Wheeler–DeWitt Equation

Unconstrained Hamiltonian Dynamics

In the Lagrangian formalism, physical histories correspond to stationary curves:

$$S = \int dt\; L(q, \dot{q}), \quad \delta S = 0.$$



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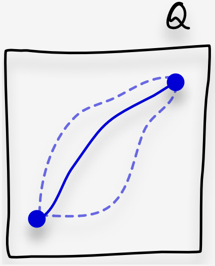

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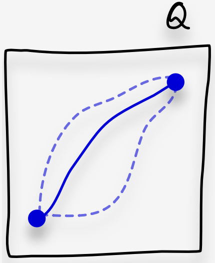
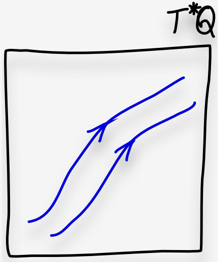
Defining: $H(p, q) := \sum_i p^i \dot{q}_i - L(q, \dot{q})$ gives Hamilton's equations:

$$\dot{q}_i = \frac{\partial H}{\partial p^i}; \quad \dot{p}^i = -\frac{\partial H}{\partial q_i}.$$





The Geometrical Perspective

$T^*\mathcal{Q}$ is a symplectic manifold (M, ω). Its structure can be used to define the Poisson Bracket:

$$\{f, g\} := \omega(X_f, X_g) = \left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p^i} - \frac{\partial f}{\partial p^i}\frac{\partial g}{\partial q_i}\right)$$

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The evolution of an arbitrary quantity, f, is given by:

$$\dot{f} = \{f, H\}.$$

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- the p^i are not independent but must satisfy “primary” constraints $\phi_n(p, q) = 0$, so
- the Legendre transformation is many-one.
- The Hamiltonian dynamics lives on a proper subspace of $T^*\mathcal{Q}$, the “constraint surface”, defined by $\phi_n(p, q) = 0$ (and any secondary constraints).

Dirac's Analysis

Dirac showed that the equations of motion can be given the following form:

$$\dot{f} = \{f, H_T\} \approx \{f, H_0\} + u^a\{f, \phi_a\}$$

where the total Hamiltonian, H_T is given by:

$$H_T(p, q, t) = H_0(p, q) + u^a(p, q, t)\phi_a(p, q)$$

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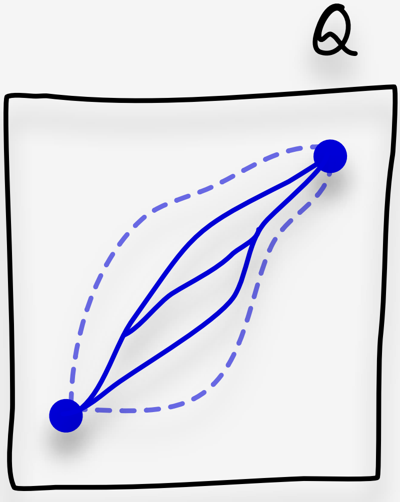
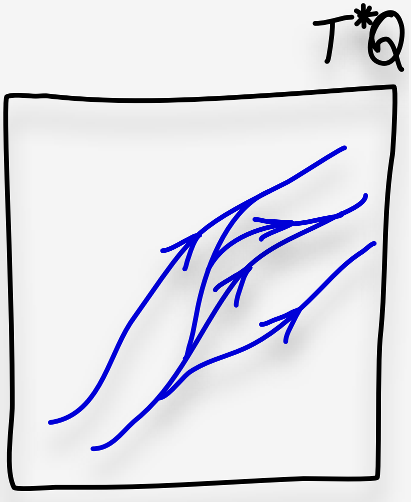
Note, the u^a are *arbitrary* functions of time.

Apparent Indeterminism





The Orthodox Interpretation

The first class constraints partition the constraint surface into “gauge” orbits.

Determinism is restored if one views:

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Geometrical interpretation

The symplectic form ω on $T^*\mathcal{Q}$ induces a presymplectic form, σ, on the constraint surface. Points lie on the same gauge orbit iff they are connected by a curve whose target vector X everywhere satisfies $\sigma(X, \cdot) = \mathbf{0}$.

Expressions of Orthodoxy

P. A. M. Dirac, who was responsible for developing the constrained Hamiltonian formalism, proposed that the gauge transformations be identified as the transformations generated by the first class constraints, where the intended interpretation is that two points of phase space which are connected by a gauge transformation are to be regarded as representing the same physical state. (Earman, 2002, 8)

Expressions of Orthodoxy

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The Assumption: [T]he results of observations and measurements must be expressed by the values of observables. (Earman, 2002, 12)

Expressions of Orthodoxy

Because of the arbitrariness of the functions λ_j in the Hamiltonian $H = h + \sum_{j=1}^{n} \lambda_j C_j$, the dynamics of the system cannot be unique. To account for this non-uniqueness one postulates that different phase space points x_1, x_2 describe the same physical state if they are connected by a gauge transformation.

Here a gauge transformation is a transformation which is generated by the constraints C_j. (Dittrich, 2007, 1894)

The Problem of Time

This leads to the (classical) Problem of Time

Plan

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Challenge 1

Pitts, J. B. (2014). “A first class constraint generates not a gauge transformation, but a bad physical change: The case of electromagnetism.” *Annals of Physics* **351**, 384–406.

Hamiltonian Electromagnetism

- $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
- $H_c = \int \mathrm{d}\mathbf{x} \left[\frac{1}{2} (\vec{\pi}^2 + \vec{B}^2) + \vec{\pi} \cdot \nabla A_0 \right]$

π^μ are the variables canonically conjugate to A_μ. Defined in terms of the Lagrangian, $\pi^\mu = -F^{0\mu}$, so:

- $\vec{\pi}$ is the electric field, and
- $\pi^0 \approx 0$ is a primary constraint.

Stability of this constraint under the Hamiltonian dynamics leads to a secondary constraint:

- $\dot{\pi}^0 = \{ \pi^0, H_c \} = \nabla \cdot \vec{\pi} \approx 0.$

Both constraints are first class.

Pitts (2014) FCCs Generate Bad Physical Changes

The π^{μ} are left unchanged.

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Transformations generated by π^0

$$\delta A_\mu(x) = \{A_\mu(x), \int d^3 y \pi^0 \xi(t,y)\} = \delta^0_\mu \xi(t,x), \text{ so}$$

$$\delta F_{\mu\nu} = \partial_\mu \xi \delta^0_\nu - \partial_\nu \xi \delta^0_\mu, \text{ and so}$$

$$\delta F_{0n} = -\delta \vec{E} = -\partial_n \xi.$$

In general $\nabla \cdot \vec{E} = 0 \mapsto \nabla \cdot \vec{E} + \nabla^2 \xi \neq 0$.

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Transformations generated by $\nabla\cdot\vec{\pi}$

$$\delta A_\mu(x) = \{A_\mu(x), \int d^3y\pi^i_{,i}\epsilon(t,y)\} = -\delta^i_\mu\frac{\partial}{\partial x^i}\epsilon(t,x), \text{leading to}$$
$$\delta F_{0n} = -\delta\vec{E} = -\partial_n\partial_0\epsilon; \quad \nabla\cdot\vec{E} \mapsto \nabla\cdot\vec{E} + \nabla^2\dot{\epsilon}.$$

Recovering the “Gauge Generator”

In general:

$$\delta A_\mu(x) = \{A_\mu(x), \int d^3y[\pi^0\xi(t,y) + \pi^i_{,i}\epsilon(t,y)]\} = \delta^0_\mu\xi - \delta^i_\mu\frac{\partial}{\partial x^i}\epsilon,$$

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One obtains $\delta F_{0n} = 0$ by setting $\xi = -\dot{\epsilon}$.
This is the form taken in electromagnetism of the “gauge generator”, discussed in the work of Castellani, and Pons, Shepley and Salisbury.
One has, for $G = \int d^3x(\pi^i_{,i}\epsilon - \pi^0\dot{\epsilon})$:

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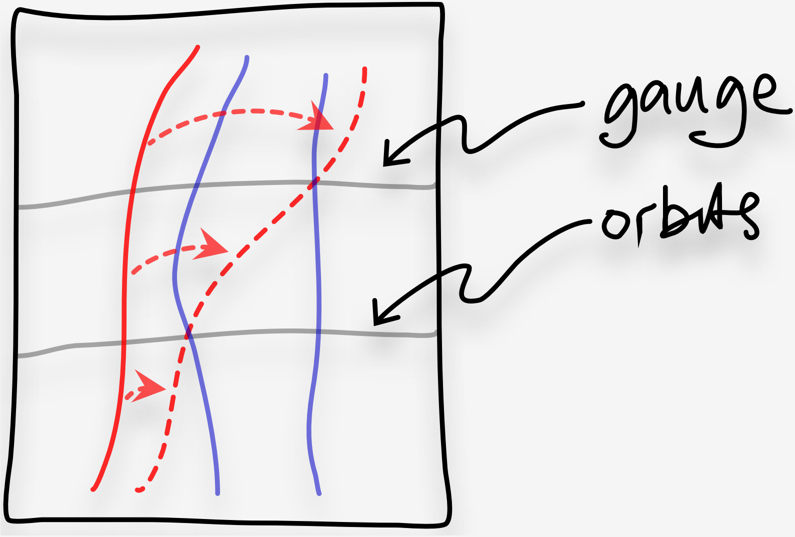
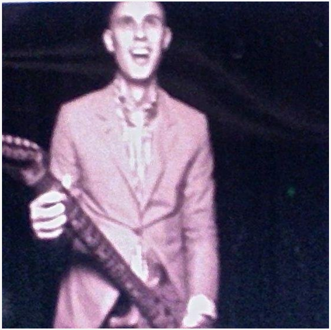
Does this undermine orthodoxy?

Orthodoxy Unscathed



Challenge 2



Barbour, Julian, and Brendan Z. Foster. “Constraints and gauge transformations: Dirac’s theorem is not always valid.” arXiv preprint arXiv:0808.1223 (2008).

Dirac's Argument

Consider the infinitesimal change in some quantity g after a short time δt.

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\begin{aligned}
g(\delta t) &= g_0 + \dot{g}\delta t \\
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But the v's are arbitrary. With different functions v' we get different $g(\delta t)$'s, where:

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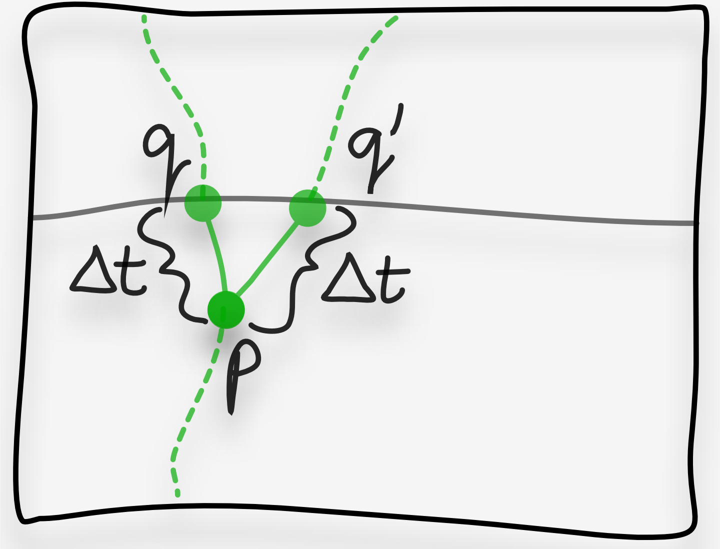
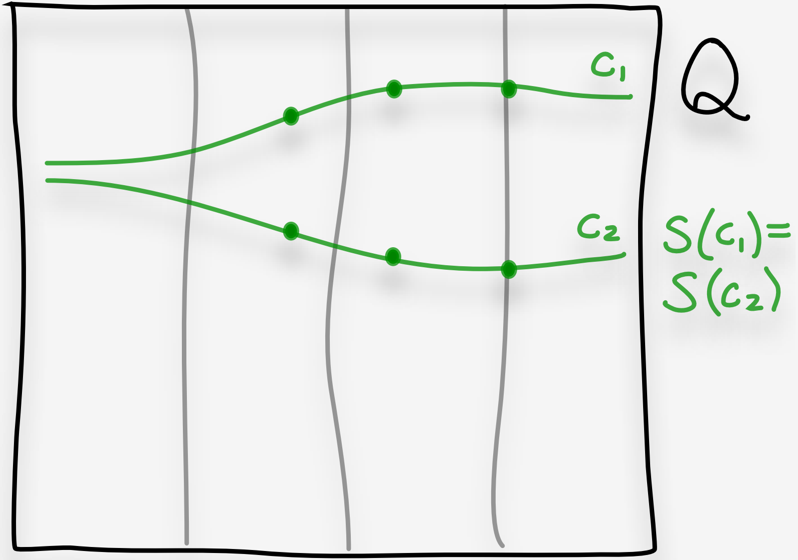


Varieties of Gauge Redundancy

Configuration space redundancy



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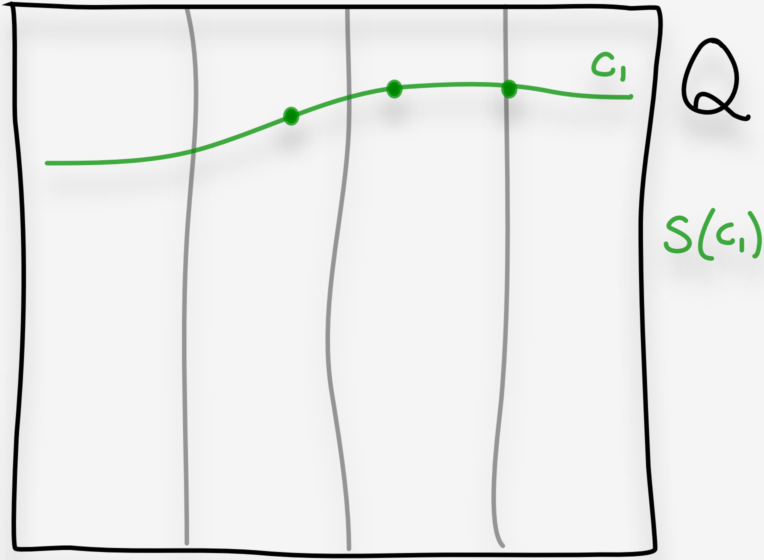
Reparameterization Invariance



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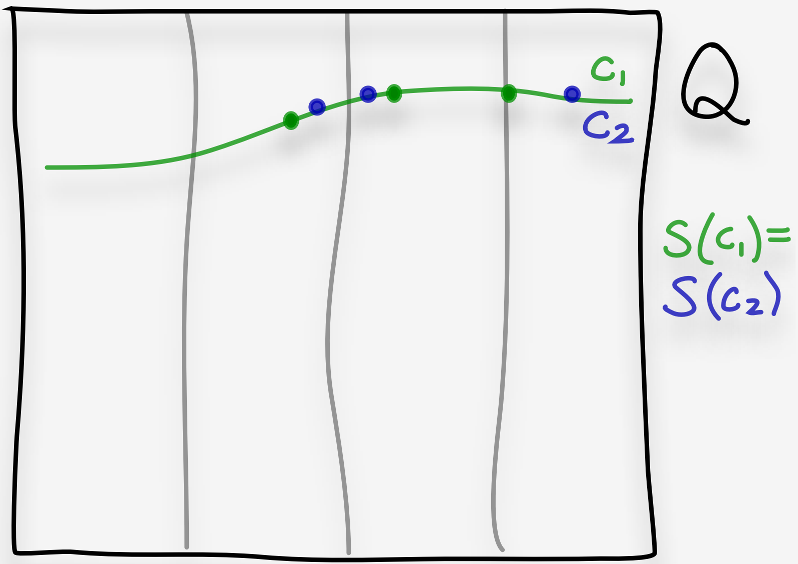

Moral: one need not identify "gauge-related" points

Finally, a word of caution. The arguments leading to the identification of [the first class constraints] as generators of transformations that do not change the physical state at a given time implicitly assume that the time t…is observable. That is information brought in from the outside. One may also take the point of view that some of the gauge arbitrariness indicates that the time itself is not observable. This is done in so-called generally covariant theories…One of the arbitrary functions is then associated with reparametrizations $t \to f(t)$ of the time variable.

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A Simple Reparamatrization Invariant Theory

Jacobi's Principle:

$$I_J = \int_A^B L_J = 2\int_A^B d\lambda \sqrt{(E-V)T}$$

The Hamiltonian is given by:

$$H = \sum_i \mathbf{p}_i \cdot \dot{\mathbf{q}}_i - L_J = Nh, \text{ where}$$

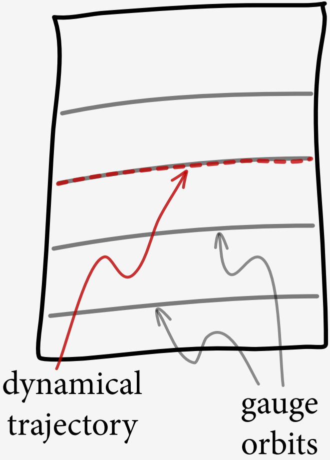
$$N = \sqrt{\frac{T}{E-V}} \quad \text{and} \quad h = \frac{1}{2}\sum_i \mathbf{p}_i \cdot \mathbf{p}_i + V - E$$

Given the definition of $\mathbf{p}_i$,

$$\mathbf{p}_i = \dot{\mathbf{q}}_i / N \qquad h \approx 0$$

h is a primary first-class constraint.

A Simple Reparamatrization Invariant Theory



- Dynamical trajectories are integral curves of X_h, where $\sigma(X_h, \cdot) = \mathrm{d}h$, but $h \approx 0$.
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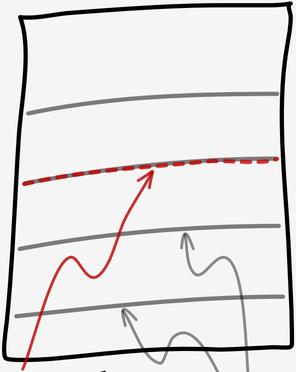

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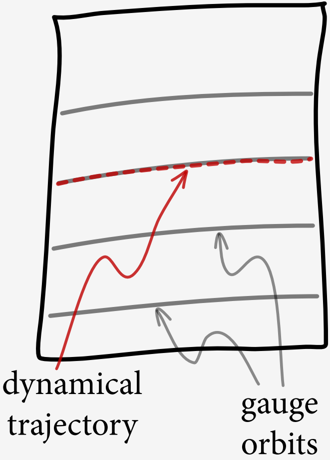

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- Distinct X_h define the same paths.
- There is no apparent indeterminism at the level of paths
  - No gauge redundancy in functions *on phase space*.
- Only the evolution of such quantities as functions of *parameter time* is undetermined.

Types of Gauge Transformation: Recap

- Maps from points to physically equivalent points
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There is no pressure (from the requirement of determinism) to require that genuine physical magnitudes weakly commute with the constraints that generate the latter.

Unfortunately there can be a third type of gauge redundancy: maps that map paths to distinct, but physically equivalent, paths without mapping points to physically distinct points.

The problem of refoliation invariance.

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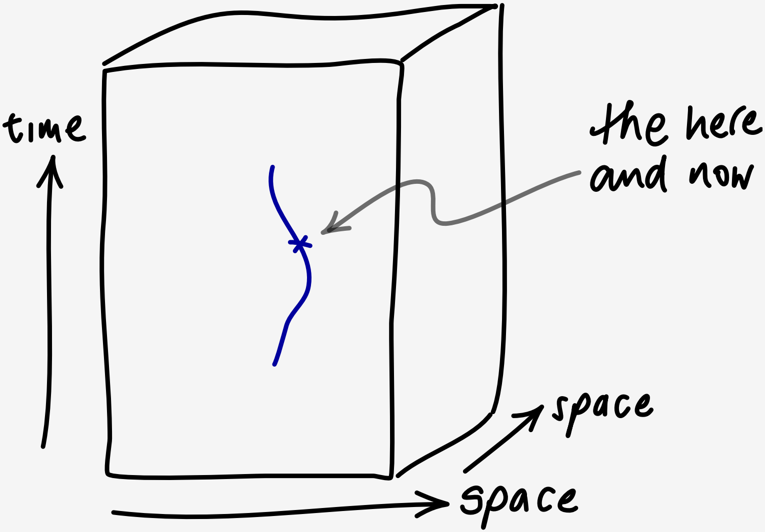
The Block Universe



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- Physical states are those invariant under transformations generated by the quantum constraints.
- Which notion of gauge does this presuppose?

The Wheeler–DeWitt Equation

- Apply Dirac constraint quantization (as summarised by Pashby) to GR
- Physical states are those invariant under transformations generated by the quantum constraints.
- Which notion of gauge does this presuppose?
- The result…

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What's wrong with a naively temporal (Everettian) understanding of transition probabilities between components of $|\Psi\rangle$?

The Proposal

it has been claimed that although the problem of time in GTR is not a pseudo-problem, neither is it intractable since common sense B-series change can be described in terms of the time independent correlations between gauge dependent quantities which change with time. (Earman, 2002, 15)

Earman on Coincidence Quantities

it remains a bit obscure how the value of this coincidence observable is measured. For if the parametrized description is taken seriously, the measuring procedure cannot work by verifying that the coincidence of values described in the equation for $X_{\bar{y}}$ does in fact take place by separately measuring the values of the clock variable and the oscillator position and then checking for the coincidence. (Earman, 2002, 13)

Against the Proposal

The problem is that all of our observations must be expressed in terms of the physically measurable quantities of the theory, namely those combinations of the dynamic variables which are [gauge invariant and therefore] independent of time. One cannot try to phrase the problem by saying that one measures the gauge dependent variables, and then looks for time independent correlations between them, since the gauge dependent variables are not measurable quantities within the context of the theory. (Unruh, 1991, 266)

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