Time as Unfolding of Process

David Rideout

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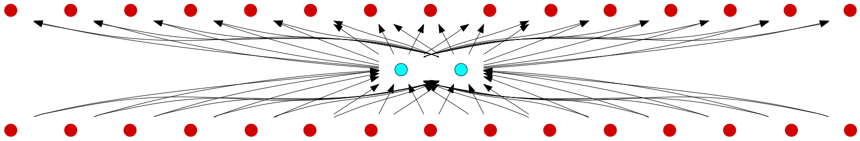
Time in Quantum Gravity 13 March 2015



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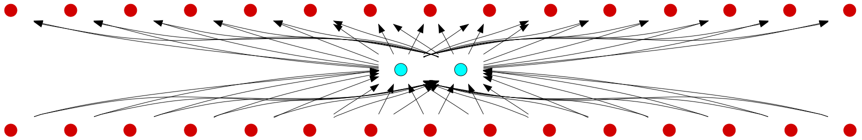


Plan of this Talk

Outline

G. 't Hooft, “Quantum gravity: a fundamental problem and some radical ideas", in *Recent developments in gravitation*, Cargèse Summer School Lectures 1978

- take lattice discretization seriously, as model for gravity
- metric tensor difficult to define on lattice, while respecting covariance
- simpler, invariant concept: causal ordering
- in discrete language, partial ordering
- this partial order *defines* the lattice
- consistency with causal ordering of continuum defines correspondence between discrete and continuum
- proposes that the early universe will emerge from a single discrete element, followed by a tree

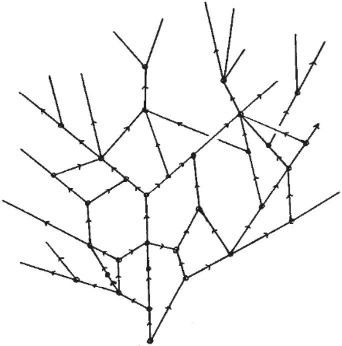
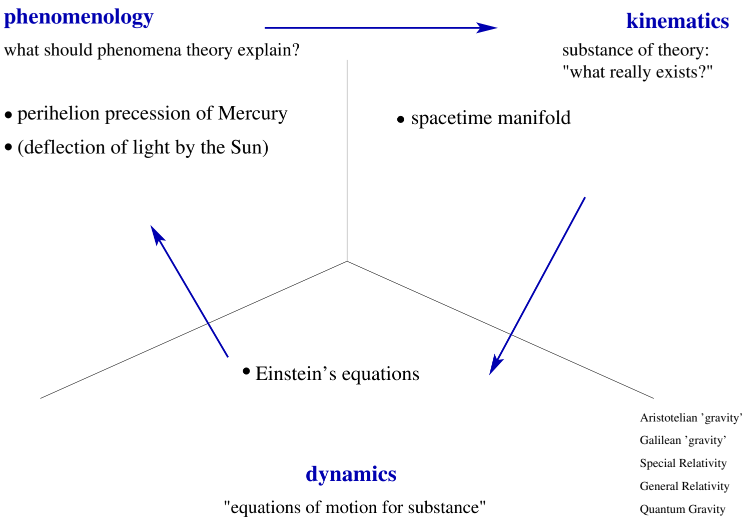


Fig. 10 The Cosmic Tree.

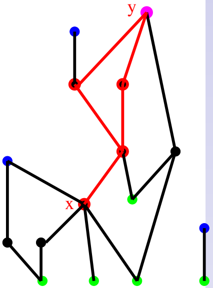
Taketani stages of theory construction



Causal Sets: Fundamentally Discrete Gravity

Based upon two main observations:

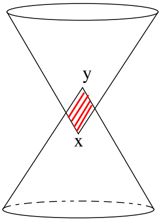
- Richness of causal structure
- Need for discreteness



Properties of discrete causal order $\prec$:

- irreflexive ($x \not\prec x$)
- transitive ($x \prec y$ and $y \prec z \Rightarrow x \prec z$)
- locally finite ($|\{y | x \prec y \prec z| < \infty$)

Some definitions:



- relation & link
- chain & antichain, height & width
- causal interval or order interval
- maximal & minimal elements

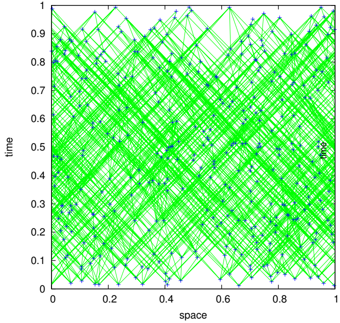
Spacetime Manifold as Emergent Structure

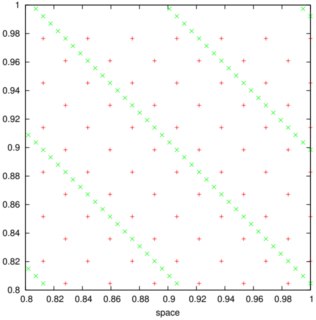
The continuum approximation

- *Embedding* – order preserving map $\phi : \mathcal{C} \to (M, g)$

$$x \prec y \Leftrightarrow \phi(x) \prec \phi(y) \;\; \forall x, y \in \mathcal{C}$$

- *Faithful embedding (‘Sprinkling’)*:
  - “preserves number – volume correspondence”
  - Spacetime does not possess structure at scales smaller than discreteness scale
- $\exists$ faithful embedding $\implies$ (M, g) approximates $\mathcal{C}$





Clock Time: Timelike Distance

Timelike geodesic is extremal chronological curve

Lorentzian signature $\Longrightarrow$ *longest* curve

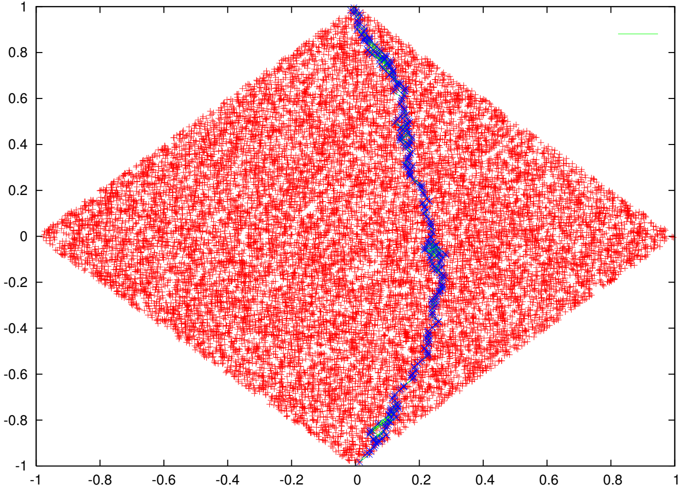
Length L of *longest* chain between x and y

$$d(x,y) := L$$

Clock Time: Timelike Distance

Length L of *longest* chain between x and y

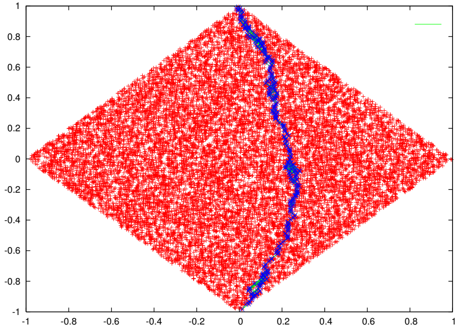
$$d(x,y) := L$$



Clock Time: Timelike Distance

Length L of *longest* chain between x and y

$$d(x, y) := L$$



Brightwell & Gregory (Phys. Rev. Lett. 66: 260-263 (1991)) state:

$$L(\rho V)^{-1/d} \to m_d \text{ as } \rho V \to \infty$$

Outline

What do we mean by Dynamics?

- Hamiltonian evolution
- Lagrangian $\rightsquigarrow$ Action
- Sum over histories
  Classical:

$$P(E) = \sum_{\gamma \in E} p(\gamma)$$

  Quantum:

$$A(\gamma) = e^{iS(\gamma)/\hbar}$$

$$P(E) = \sum_q \Big| \sum_{\gamma \in E; \gamma(T) = q} A(\gamma) \Big|^2$$

- Quantum Measure

$$P(E) = D(E, E)$$

$$D(E_1, E_2) = \text{“} \sum_{\gamma \in E_1, \gamma' \in E_2} e^{i(S(\gamma) - S(\gamma'))/\hbar} \delta(\gamma(T), \gamma'(T)) \text{”}$$

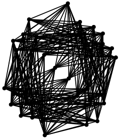
Simplest Dynamical Law: ‘Typical’ Objects

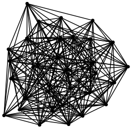
Simplest Dynamical Law: ‘Typical’ Objects

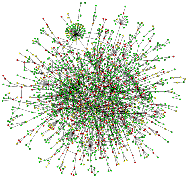
Sequence of many coin flips.
Which is the ‘typical’ sequence?

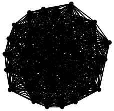
1. HTHHHHTTHHTHTHHTTTTTHTHHTTTHHHHTTTHHHTH
2. HHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHH
3. THTHTHTHTHTHTHTHTHTHTHTHTHTHTHTHTHTHTHT
4. HTHTTHTTTHTTTTHTTTTTHTTTTTTHTTTTTTTHTTT

Typical Graphs

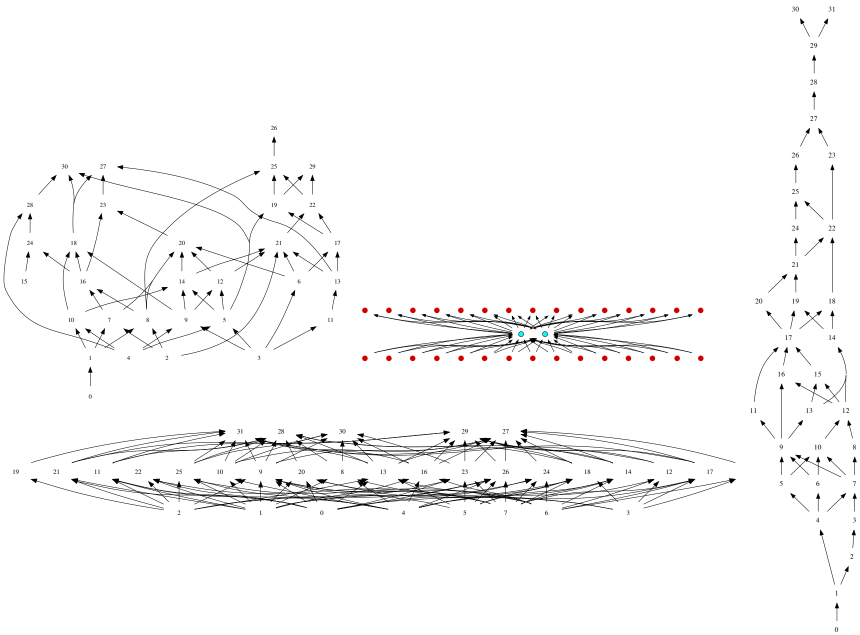




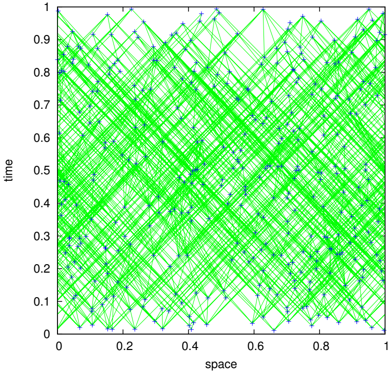


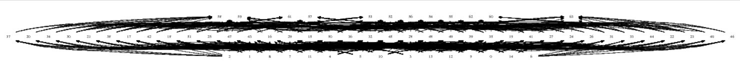


Typical Causal Sets



‘Entropy Crisis:’ Dynamical Emergence of the Continuum





$2^{N \ln N}$ continuua vs. $2^{N^2/4}$ Kleitman-Rothschild orders

Sequential Growth Dynamics

DR, R Sorkin

Grow causal set, ‘one element at a time’, beginning with empty set
Stochastic (Markov) process
Probabilities based upon three principles

- ‘Internal temporality’ (Causet grows only to the future)
- Discrete general covariance
- Bell causality

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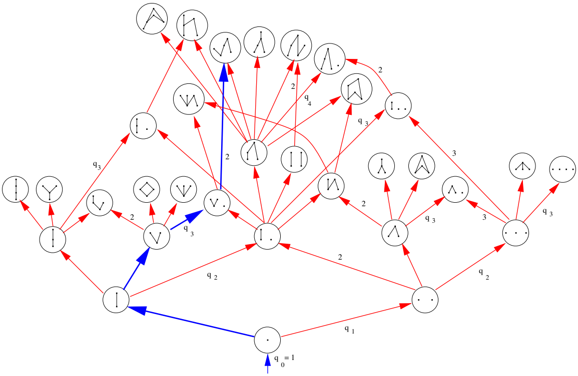
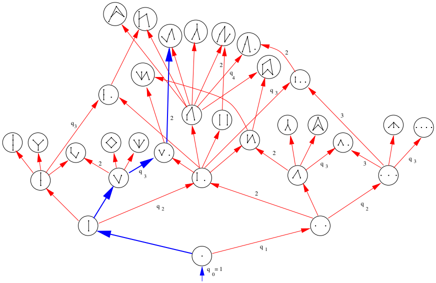


The poset of finite causal sets

Sequential Growth Dynamics

DR, R Sorkin



The poset of finite causal sets

$$\Pr(C_n \to C_{n+1}) \propto \sum_{k=0}^{\varpi - m} \binom{\varpi - m}{k} t_{k+m}$$

- Infinite sequence of free parameters (‘coupling constants’) $t_n \geq 0$
- ‘Transitive percolation’ dynamics $t_n = \left(\frac{p}{1-p}\right)^n$

Outline

Time as Unfolding of (a Quantum) Process: Results

'Pre-Quantum' :

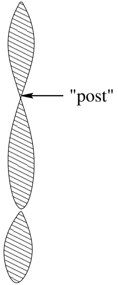
- Cyclic cosmology with evolving 'coupling constants'
- Gives rise to deSitter like early universe
- Gives rise to 'internal time' within Complex Networks (*e.g.* Internet)

Quantum :

- When do the Kleitman-Rothschild causets dominate? (in volume-time, n)
- Are almost all histories roughly time-reversal symmetric?
- Is the dynamics able to escape from the Kleitman-Rothschild super-exponential dominance?
- Is there current observational evidence hinting at quantum cosmology of this form?

Structure of Transitive Percolation

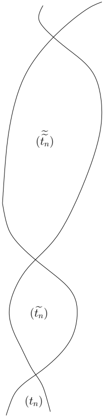
- Completely homogeneous : future of an element is independent of anything spacelike to it
- Due to random fluctuations, however, appears inhomogeneous in time



- 'Originary' dynamics subsequent to post : Each newborn element must connect to at least one other element
- Universe expands to volume $\sim 1/p$

Cosmic Renormalization

- Growth dynamics formally Markovian, because entire past history is taken as current state, however has long memory
- ‘Cosmic renormalization’:
  Can describe growth of subsequent cycles as new (originary) dynamics, with renormalized parameters (t_n)
- Transitive percolation is unique fixed point of cosmic renormalization
- Attractive fixed point, no cycles (pointwise convergence)
- Known that $t_n = (\alpha/\ln n)^n$, $\alpha \geq \pi^2/6$ contains infinite number of posts.



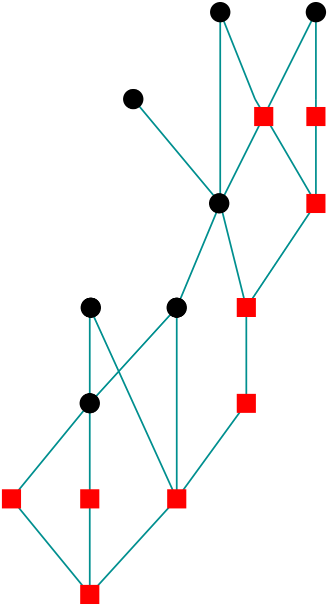
(Denjoe O’Connor, Xavier Martin, DR, Rafael Sorkin)
(Avner Ash, Patrick McDonald, Graham Brightwell)

Originary Percolation

Random tree era

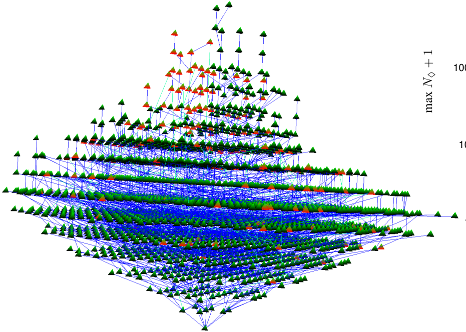
- limit $p \ll 1$
- originary — each elt chooses exactly one ancestor $\rightsquigarrow$ simple model of random tree
- exponential expansion
- future of every element itself originary percolation $\implies$ causal set is 0+1 dimensional at smallest scales
- not exactly spacetime manifold of GR

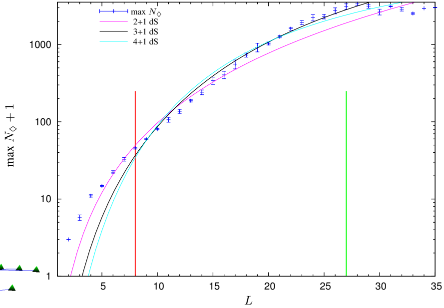
Originary Percolation



$N = 16, p = 0.2$

Early Universe of Growth Dynamics



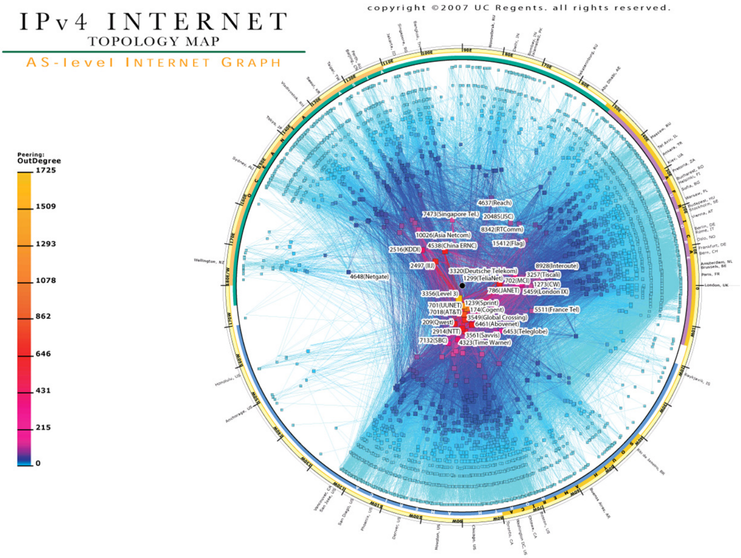


$$\ell = 6.81 \pm .72$$

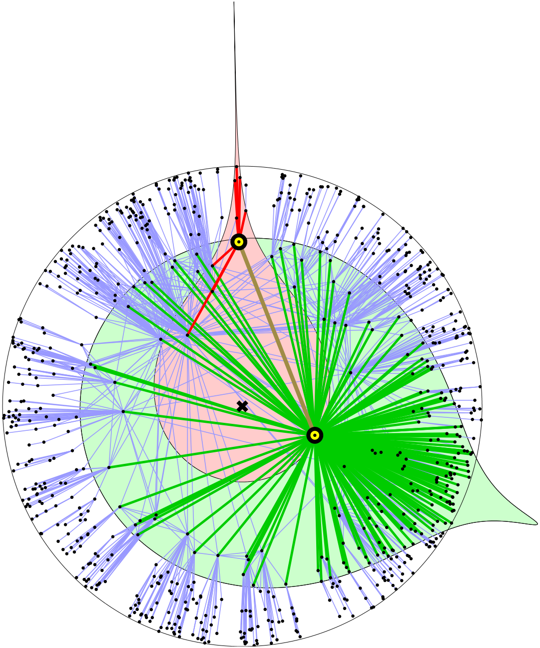
$$m = 1.926 \pm .023$$

M. Ahmed and DR, *Phys.Rev.D* **81**, 083528 (2010)
arXiv:0909.4771 [gr-qc]

Network Science



Network Science



Network Science

SCIENTIFIC REPORTS

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Network Cosmology

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SUBJECT AREAS:

STATISTICAL PHYSICS, THERMODYNAMICS AND NONLINEAR DYNAMICS

THEORETICAL PHYSICS

APPLIED PHYSICS

COSMOLOGY

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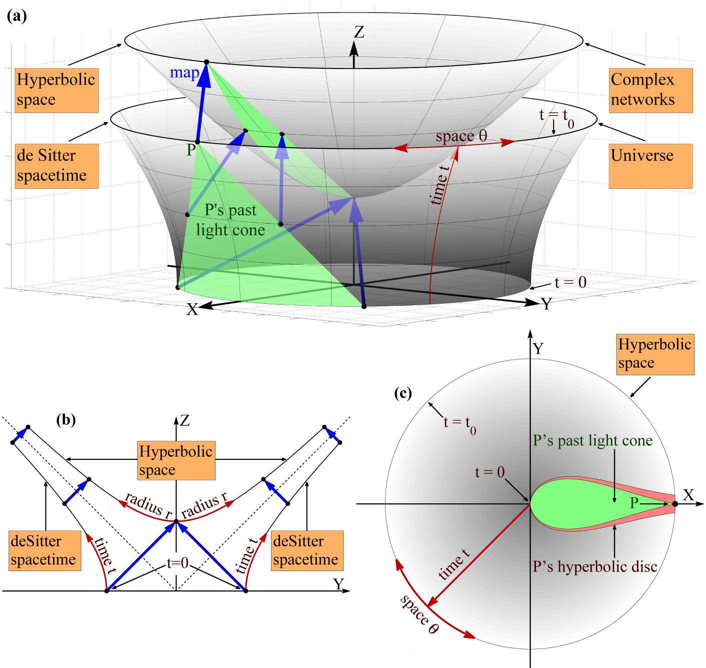
Prediction and control of the dynamics of complex networks is a central problem in network science. Structural and dynamical similarities of different real networks suggest that some universal laws might accurately describe the dynamics of these networks, albeit the nature and common origin of such laws remain elusive. Here we show that the causal network representing the large-scale structure of spacetime in our accelerating universe is a power-law graph with strong clustering, similar to many complex networks such as the Internet, social, or biological networks. We prove that this structural similarity is a consequence of the asymptotic equivalence between the large-scale growth dynamics of complex networks and causal networks. This equivalence suggests that unexpectedly similar laws govern the dynamics of complex networks and spacetime in the universe, with implications to network science and cosmology.

Physics explains complex phenomena in nature by reducing them to an interplay of simple fundamental laws.

Network Science



Outline

Path Integral for Gravity

- Lorentzian functional integral for gravity

$$Z = \int_{(M,g)} e^{iS_{\rm EH}[(M,g)]/\hbar}$$

- Lorentzian 'path' sum over causal sets

$$Z = \sum_C e^{iS_{\rm EH}[C]/\hbar}$$

- Restract sum to fixed (finite) cardinality
  $\longrightarrow$ Fixed spacetime volume $\sim$ unimodular gravity
- Need expression for $S_{\rm EH}[C]$

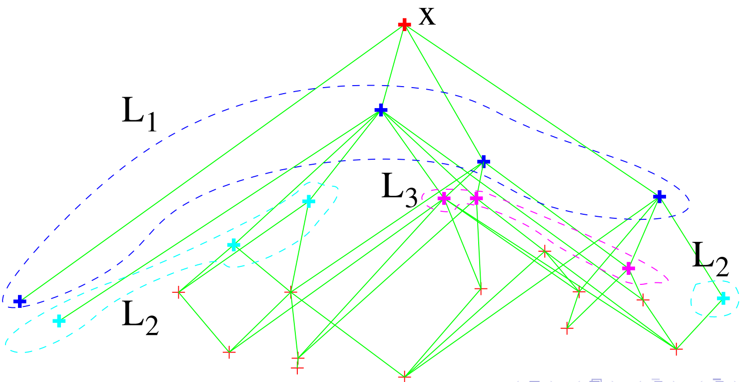
Discrete $\Box$: Towards an Expression for $S_{\mathrm{EH}}[C]$

- Discrete D'Alembertian operator (R. Sorkin)

$$\Box^{(2)}\phi(x) = \frac{4}{\ell^2}\left(-\frac{1}{2}\phi(x) + \left(\sum_{y\in L_1} - 2\sum_{y\in L_2} + \sum_{y\in L_3}\right)\phi(y)\right)$$

where

$$L_i = \{y \in C | y \prec x \ \text{ and } \ N_\Diamond(y,x) = i-1\}$$



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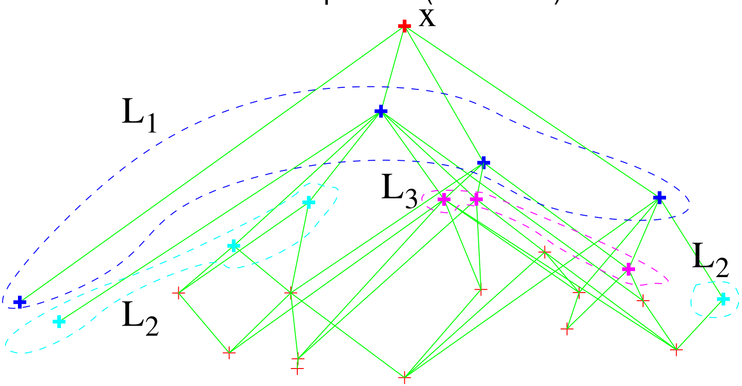
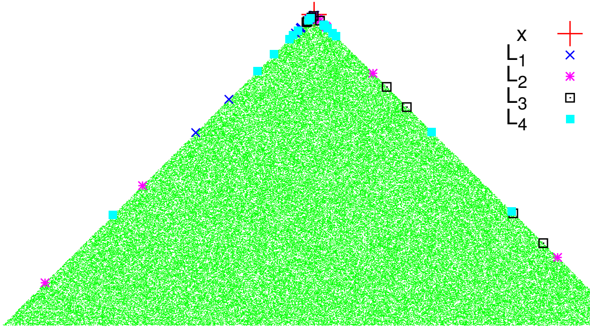


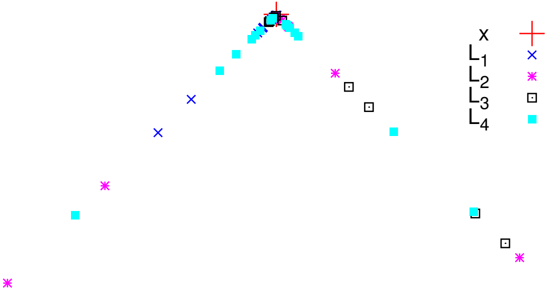
- In 4d: (Benincasa-Dowker)

$$\Box^{(4)}\phi(x) = \frac{4}{\sqrt{6}\ell^2}\left(-\phi(x) + \left(\sum_{y\in L_1} -9\sum_{y\in L_2} +16\sum_{y\in L_3} -8\sum_{y\in L_4}\right)\phi(y)\right)$$

High density $\ell \to 0$ limit



High density $\ell \to 0$ limit



Einstein-Hilbert action for Causal Sets

(Benincasa-Dowker PRL Jan 2010))

- In curved spacetime:

$$\lim_{\ell \to 0} \langle \Box^{(4)} \phi(x) \rangle = \left(\Box - \frac{1}{2} R(x) \right) \phi(x)$$

- $\Box^{(4)}$ (-2) gives Ricci scalar
- Use to write Einstein-Hilbert action for causal set

$$S_{EH}^{(4)}[C] = O(1)(N(C) - N_1(C) + 9N_2(C) - 16N_3(C) + 8N_4(C))$$

where $N_i(C) = |\{x, y \in C | N_\Diamond(x, y) = i - 1\}|$

- Expression for path sum for causal sets, appropriate to 4d:

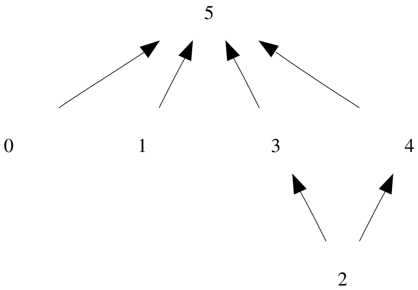
$$Z = \sum_{C \in \mathcal{C}} e^{i\widetilde{\beta}(N(C) - N_1(C) + 9N_2(C) - 16N_3(C) + 8N_4(C))}$$

Einstein-Hilbert action for Causal Sets

(Benincasa & Dowker PRL Jan 2010)

- Expression for (4d) path sum for causal sets:

$$Z = \sum_{C \in \mathcal{C}} e^{iS_{\mathrm{EH}}[C]/\hbar} = \sum_{C \in \mathcal{C}} e^{i\widetilde{\beta}(N(C) - N_1(C) + 9N_2(C) - 16N_3(C) + 8N_4(C))}$$



$$N = 6$$
$$N_1 = 6$$
$$N_2 = 0$$
$$N_3 = 1$$

Generalized 'Wick Rotation'

- Usual approach is to perform Wick rotation $t \to it$
- Alternative: Analytically continue coefficient $\widetilde{\beta} \mapsto i\beta$
  Casts sum into thermodynamic partition function

$$Z = \sum_{C \in \mathcal{C}} e^{-\beta(N(C) - N_1(C) + 9N_2(C) - 16N_3(C) + 8N_4(C))}$$

- $\rightsquigarrow$ 'Euclidean' sum, can be analyzed numerically using Metropolis Monte Carlo techniques

Markov Chain Monte Carlo on Causal Sets

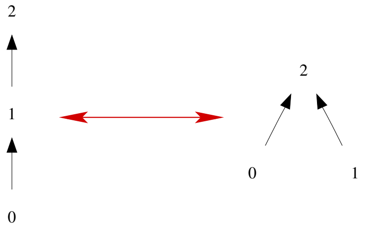
(J. Henson, DR, R. Sorkin, S. Surya)

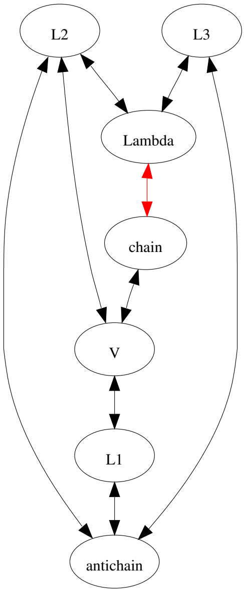
- Markov Chain: random walk on set of ‘states’, governed by mixing matrix M
- Theorem: If M satisfies
  - Ergodicity
  - Detailed balance

$$\mathrm{Pr}(C_1)\mathrm{Pr}(C_1 \to C_2) = \mathrm{Pr}(C_2)\mathrm{Pr}(C_2 \to C_1)$$

  then, independently of initial state, at late times probability to visit state C is $\mathrm{Pr}(C)$
- Metropolis Monte Carlo over (naturally labeled) partial orders ($x \prec y \implies x < y$)
- Found two moves which satisfy these conditions $\longrightarrow$ Use uniform mixture of two moves
- Transitivity — must enforce non-local constraint on relations
- Define *link* $x \lessdot y$: $x \prec y$ and $\{z|x \prec z \prec y\} = \emptyset$

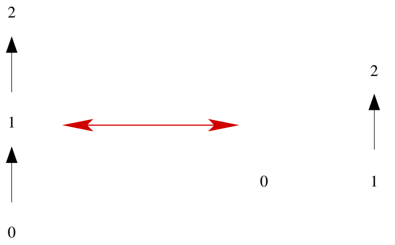
Relation Move

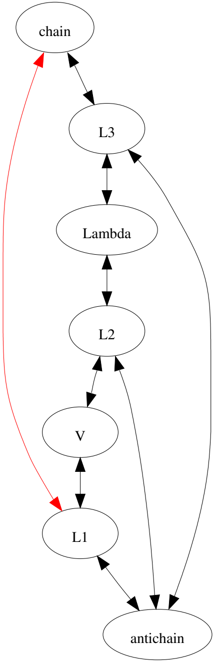




- If $x \lessdot y$: Remove single relation $x \prec y$
- If $x \not\prec y$ and form *critical pair* $(\mathrm{past}(x) \subseteq \mathrm{past}(y)$ and $\mathrm{fut}(y) \subseteq \mathrm{fut}(x))$: Insert single relation $x \prec y$
- Else do nothing

Link Move

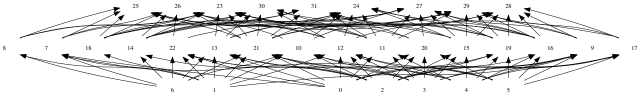




- If $x \lhd y$: Remove all relations from $\mathrm{incpast}(x)$ to $\mathrm{incfut}(y)$, save those required by transitivity via other elements
- If $x \not\prec y$, and $\nexists$ links from $\mathrm{incpast}(x)$ to $\mathrm{incfut}(y)$: Insert all relations from $\mathrm{incpast}(x)$ to $\mathrm{incfut}(y)$
- Else do nothing

$\hbar \to \infty$ Limit

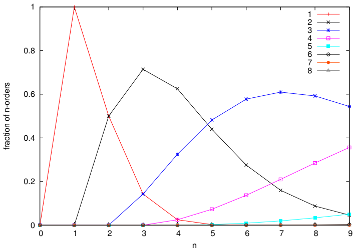
- $\hbar \to \infty \implies \beta = 0 \rightsquigarrow$ Uniform measure on sample space
- Causal sets on up to 16 elements enumerated explicitly
- Kleitman-Rothschild theorem (Trans. AMS 1975)

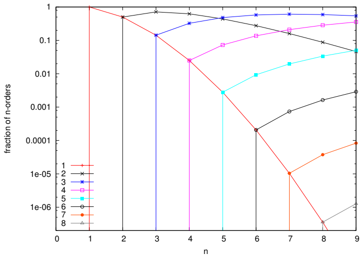


- Non-locality / long range interaction $\rightsquigarrow$ How big is big?

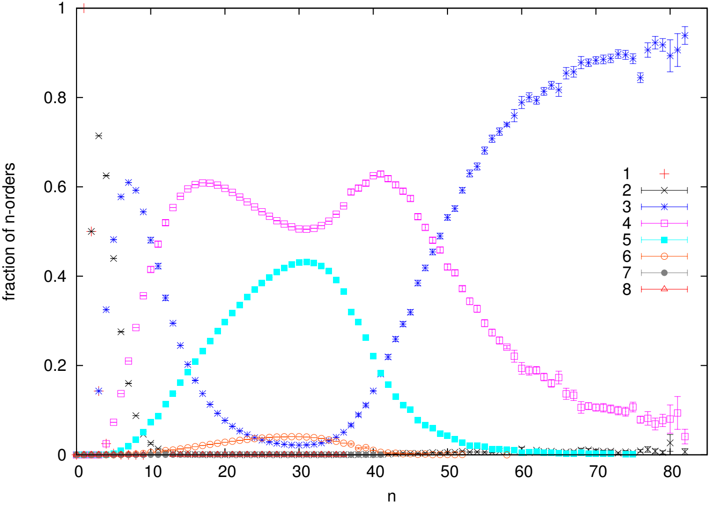
> *In the remainder of this paper we will adopt the convention that any inequality or other statement about functions of n will be meant to be true only for all n sufficiently large, where how large depends on the statement. This will be a convenience since there are so many such statements below.*

Height distribution for $n \leq 9$

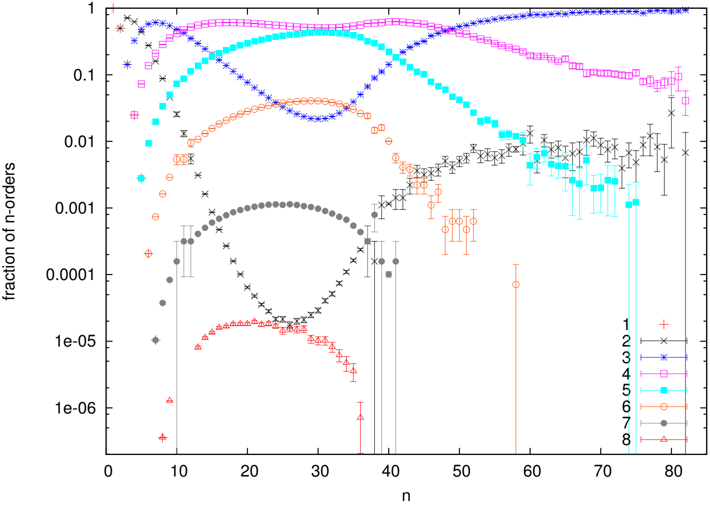




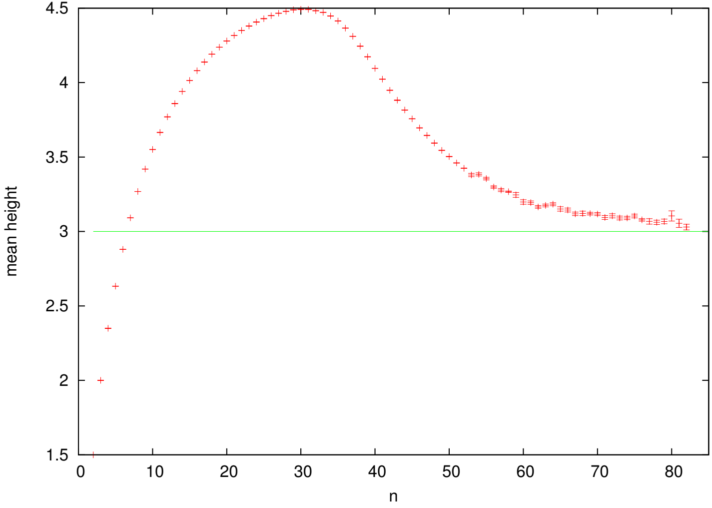
Height distribution for $n \leq 82$



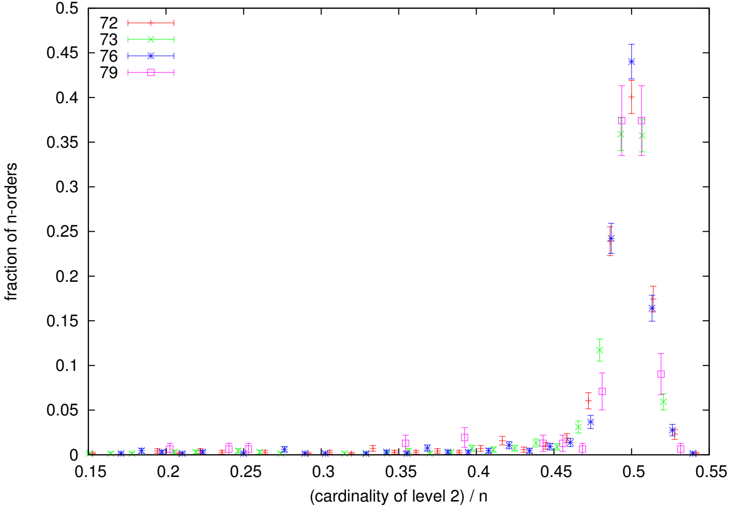
Height distribution for $n \leq 82$ (logscale)



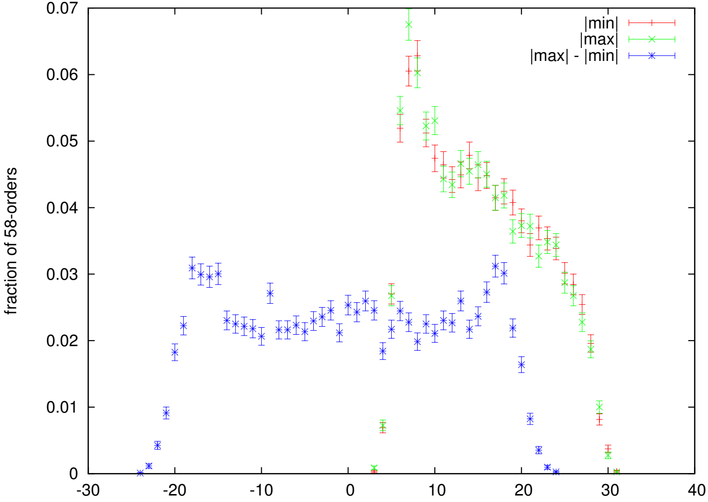
Mean height for $n \leq 82$



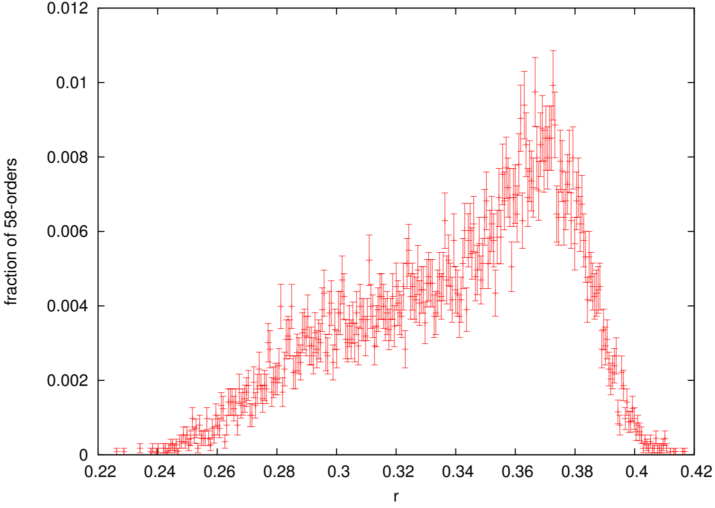
Cardinality of Level 2



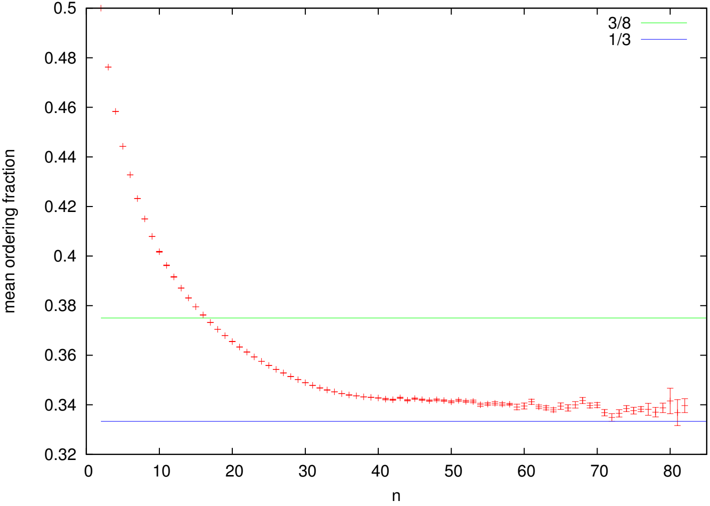
Number of minimal and maximal elements



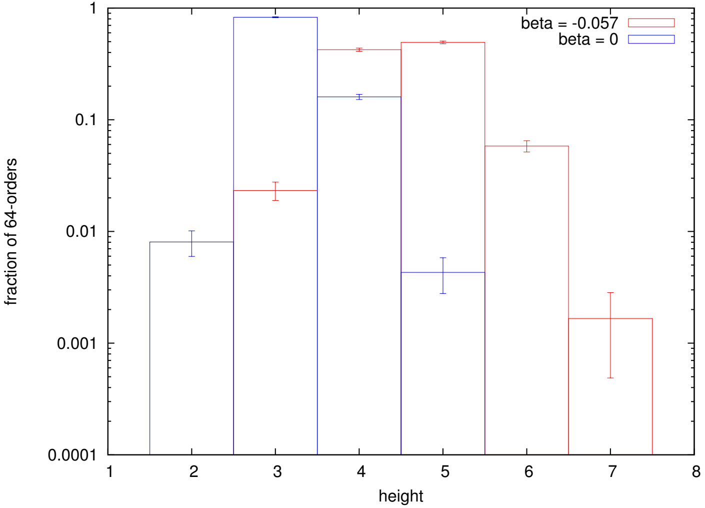
Ordering Fraction



Mean ordering fraction for $n \leq 82$



Escape from KR orders

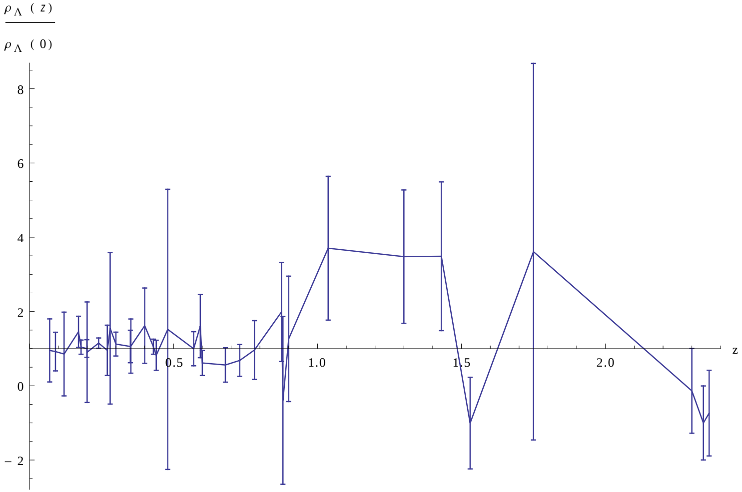


Tension with ΛCDM Concordance Model

> *"Current measurements of the low and high redshift Universe are in tension if we restrict ourselves to the standard six parameter model of flat ΛCDM."*

[Wymann, Rudd, Vanderveld, Hu, arXiv:1307.77152 (2 Jan 2014)]

Tension with ΛCDM Concordance Model



[I. Jubb, F. Dowker, private communication, based on arXiv:1407.5405 [gr-qc]]

Outline

Time as Unfolding of (a Quantum) Process

'Pre-Quantum' :

- Cyclic cosmology with evolving 'coupling constants'
- Gives rise to deSitter like early universe
- Gives rise to 'internal time' within Complex Networks (*e.g.* Internet)

Quantum :

- When do the Kleitman-Rothschild causets dominate? (in volume-time, n) $\rightsquigarrow$ For some $n > 100$ perhaps.
- Are almost all histories roughly time-reversal symmetric? $\rightsquigarrow$ No!
- Is the dynamics able to escape from the Kleitman-Rothschild super-exponential dominance? $\rightsquigarrow$ Yes!
- Is there current observational evidence hinting at quantum cosmology of this form? $\rightsquigarrow$ Perhaps!

Escape from KR orders