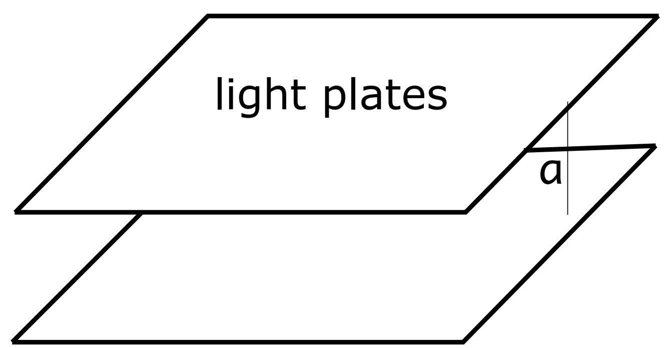
Can Quantum Mechanics Breed Negative Masses?

Bruno Arderucio, June 2022

Planar Casimir geometry *appears* promising

Regularised stress-energy tensor

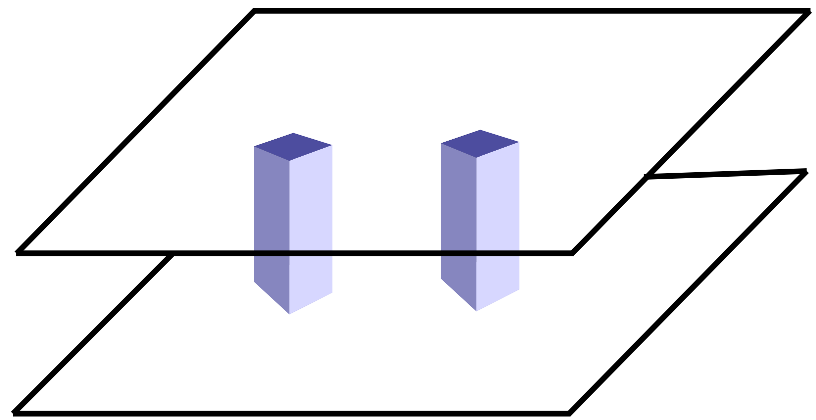
$$\langle T_{\mu\nu}\rangle = \frac{\pi^2}{720a^4}[\theta(z) - \theta(z-a)]\mathrm{diag}(-1,1,1,-3)$$



The EM field cannot exist in isolation

$\nabla^\nu \langle T_{\mu\nu} \rangle \neq 0$ at $z = 0, a$

At least, dynamical equilibrium requires struts compensating the attractive force $\langle T_{zz} \rangle \times \mathrm{area}$



Either the struts are classical or they aren't. Let's initally focus on the first alternative

Classical struts yield to overall positive mass

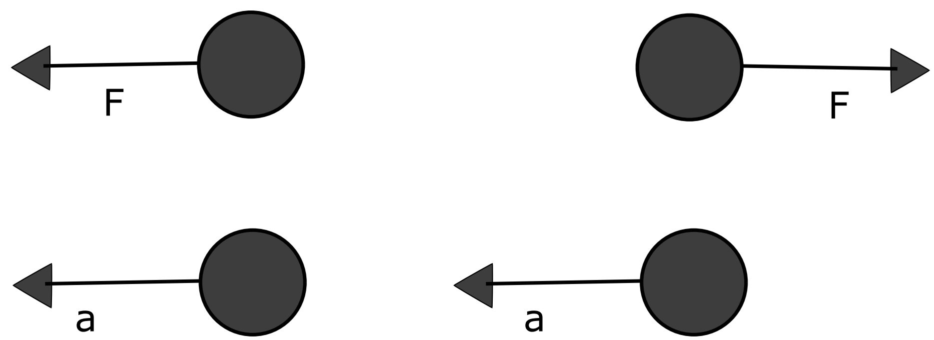
Overall Mass $M = \int \mathrm{d}^3x \ \left(\langle T_{00}\rangle + T_{00}^{\mathrm{struts}}\right) \geq \frac{\pi^2 A}{360a^3} > 0$

All the struts have to do is to balance out the total force. It is not even necessary to have $\nabla_b\left(\langle T^{ab}\rangle + T^{ab}_{\mathrm{struts}}\right) = 0$ everywhere.

In particular, the plates play no role and can be arbitrarily light.

There are general reasons to expect M>0

Run-away solutions



General expectation: stationarity + asymp. flatness + regularity
for physically realistic systems yields M>0

All these hypotheses are necessary

Morris-Thorne wormhole $\mathrm{d}s^2 = -\mathrm{e}^{2\Phi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-b(r)/r} + r^2\mathrm{d}\Omega^2$

Komar mass can have any sign: $-\lim_{r\to\infty} \Phi' \sqrt{1-\frac{b(r)}{r}}\,\mathrm{e}^{\Phi} r^2$

ADM mass $M = \lim_{r\to\infty} \frac{1}{2} \frac{b(r)}{1-b(r)/r}$

So only asymtotically flat, regular wormholes possess M>0