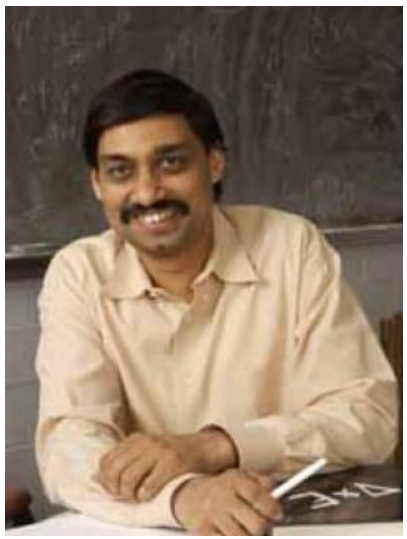
Why Black Hole Information Loss is Paradoxical

David Wallace (University of Pittburgh)

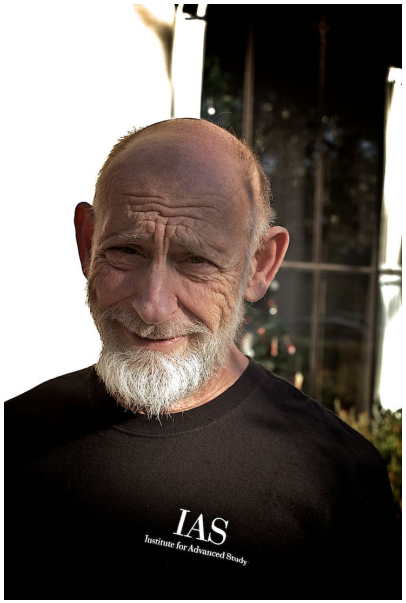
29th June 2022

Samir Mathur



We shall see that there is a very precise statement of the contradiction found by Hawking, and that bypassing the paradox needs a basic change in our understanding of how quantum effects operate in gravity.
(arxiv:0909.1038)

Leonard Susskind

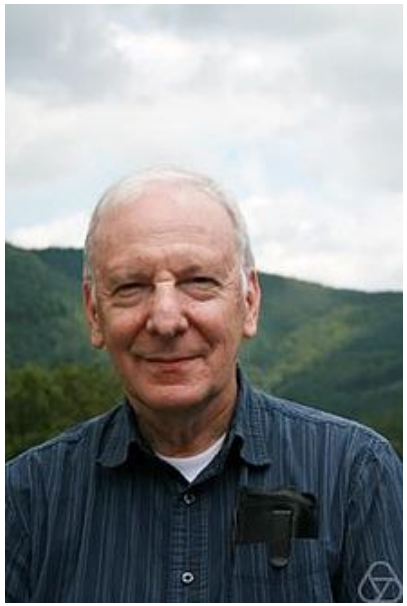


Whatever else Maldacena and Witten had done, they had proved beyond any doubt that information would never be lost behind a black hole horizon. The string theorists could understand this immediately; the relativists would take longer. (The Black Hole War)

Stephen Hawking

Because part of the information about the state is lost down the hole, the final situation is represented by a density operator rather than a pure quantum state. This means there is no S matrix for the process of black-hole formation and evaporation. (Phys Rev D 14, 2460)

Bill Unruh and Bob Wald



The proposals to [avoid information loss] typically require drastic violation of the local laws of physics . . . loss of information in black hole formation and evaporation does not violate any fundamental principle of physics and is not, in any way, a radical proposal. Thus, our strong inclination is to believe that there is loss of information in the process of black hole formation and evaporation. (arxiv:1703.02140)

Gordon Belot, John Earman, Laura Ruetsche

Assuming that black hole evaporation can be described by a spacetime of classical GR, that the evaporation is not of the thunderbolt type, and that the quantum aspects of the problem can be described by QFT on the resulting spacetime, then none of the escape routes discussed in the literature provides a plausible way to avoid the conclusion that the post-evaporation state is mixed. . . . information is lost, as Hawking originally maintained. (BJPS 50 (1999) pp.189-229)

Tim Maudlin

The so-called "information loss paradox" arises rather from the inaccurate application of foundational principles, involving both mathematical and conceptual errors. The resources for resolving the "paradox" are familiar and uncontroversial, and have been pointed out in the literature. The problem ought to have been dismissed 40 years ago. Recent radical attempts to "solve" the problem are blind alleys, solutions in search of a problem.
(arxiv:1705.03541)

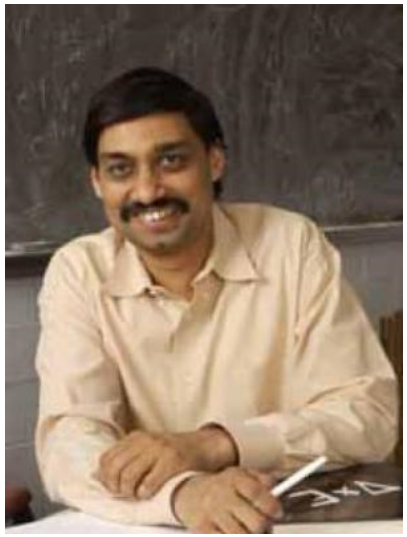
Maudlin again

How could a simple solution have gone unappreciated by so many theoretical physicists — among them very great physicists — over such a long period of time? Probably no completely satisfactory non-sociological explanation is possible. (ibid.)

Hawking again

[Q]uantum gravity is unitary and information is preserved in black hole formation and evaporation.. (arxiv:hep-th/0507171)

Mathur again



Not everyone understands Hawking's paradox the same way [.] (ibid.)

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- Completely fails to reproduce those features of thermodynamics that concern multiple thermally-interacting systems

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- Very strong theoretical evidence that it carries away energy

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$$S = 2\pi(Q_1 Q_5 N - J^2)(1+3/2N) - \frac{1}{12}(n_v - 3)\ln(Q_1 Q_5 N - J^2).$$

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- The extremal-hole calcuations can be reinterpreted as ADS_3/CFT_2 calculations

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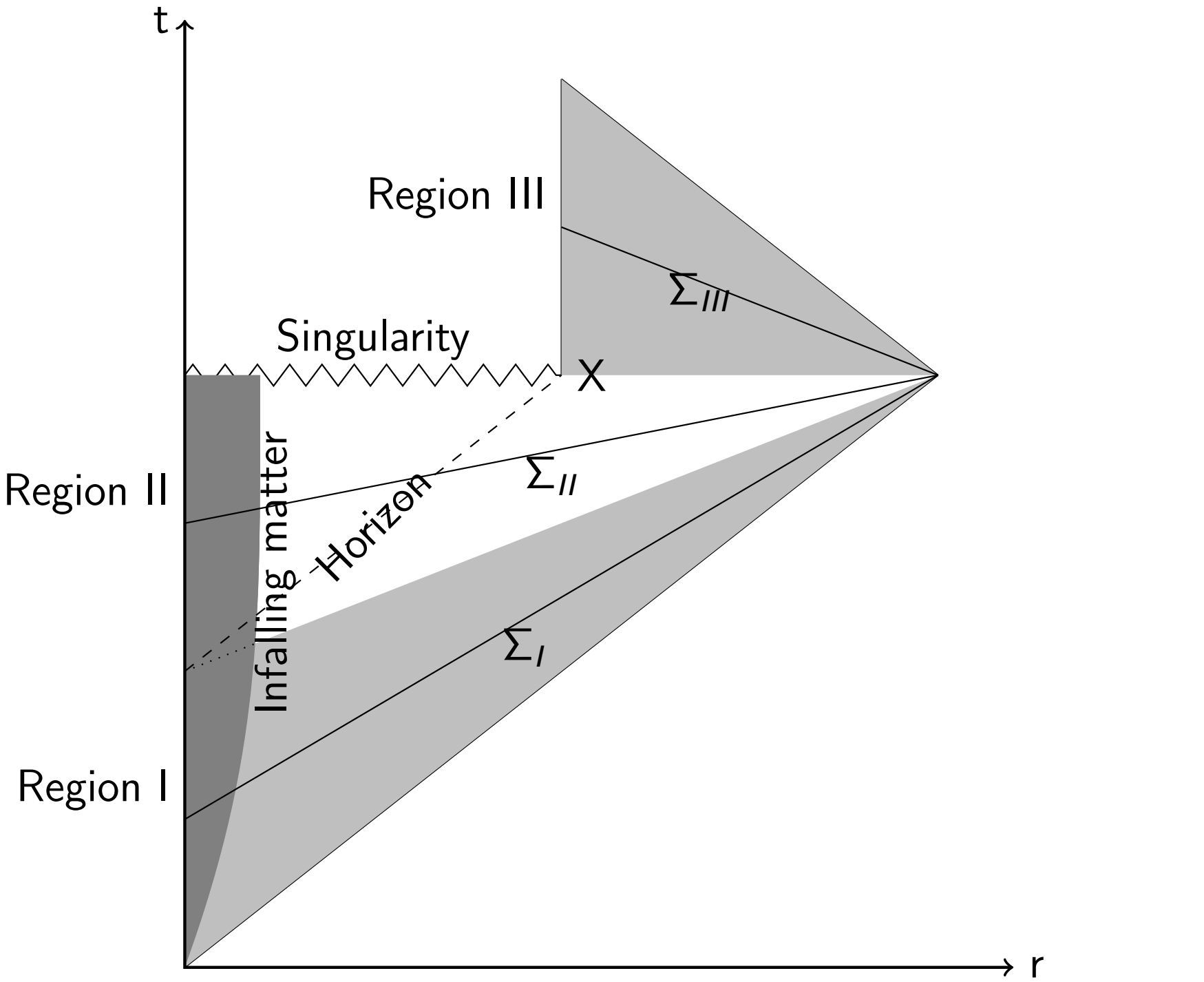
- Hawking radiation is ordinary thermal cooling of that membrane


t
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Σ_{III}
Singularity
X
Region II
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Horizon
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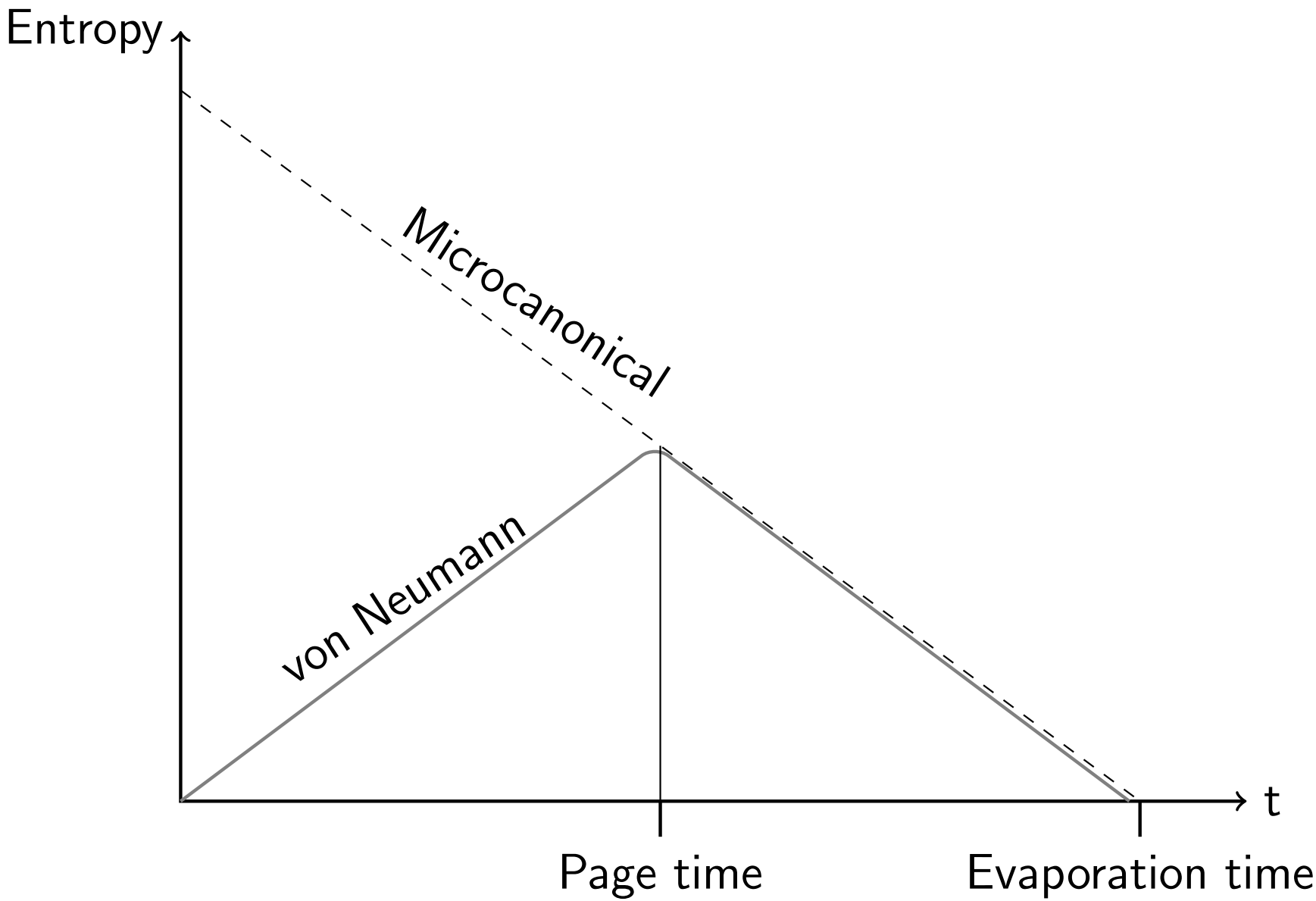

Figure: The Page curve

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- Clash between QMP and QFT

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- Extremal (charged and/or rotating) black holes, perturbed away from extremality and then allowed to decay back down again, where QMP predicts the decay radiation is in a pure state

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The string decay rates, d to the large black hole region, agree precisely with the semiclassical Hawking decay rates in a wide variety of circumstances. However, the string method not only supplies the decay rates, but it also gives a set of unitary amplitudes underlying the rates. We find it tempting to conclude that these extrapolated amplitudes are also correct. It is hard to imagine a mechanism which corrects the amplitudes, but somehow conspires to leave the rates unchanged.

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- For a small black hole in AdS space formed by infalling matter, the Poincaré recurrence theorem ensures that the BH will eventually decay completely.

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- Monogamy of entanglement makes these two requirements inconsistent

Daniel Harlow

Thus we find ourselves in the enviable situation of having an interesting problem with no really satisfying answer; if we are lucky this means that we will learn something deep. (arxiv:1409.1231)