The path to

Quantum Gravity

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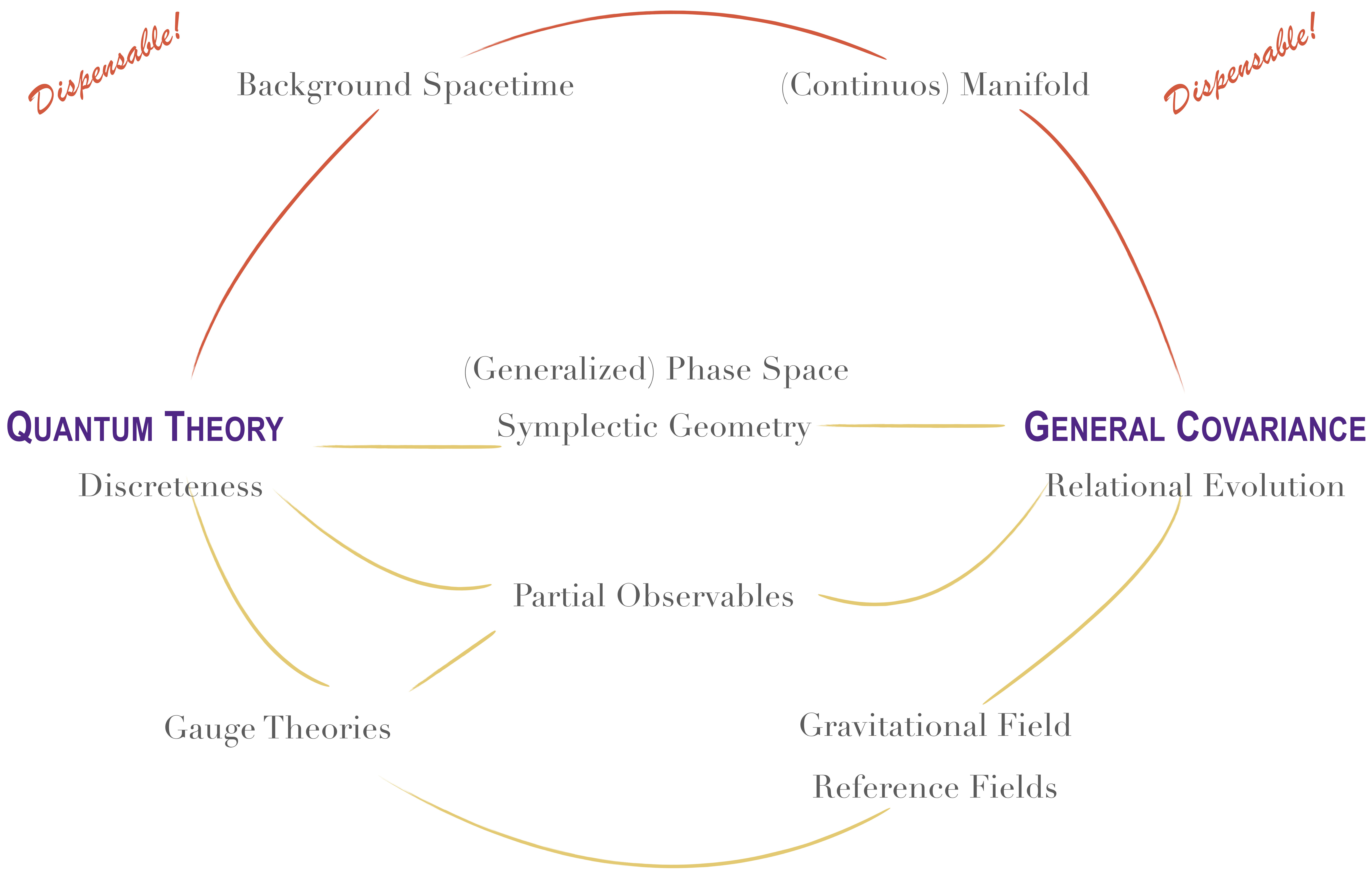
QUANTUM GRAVITY GOAL

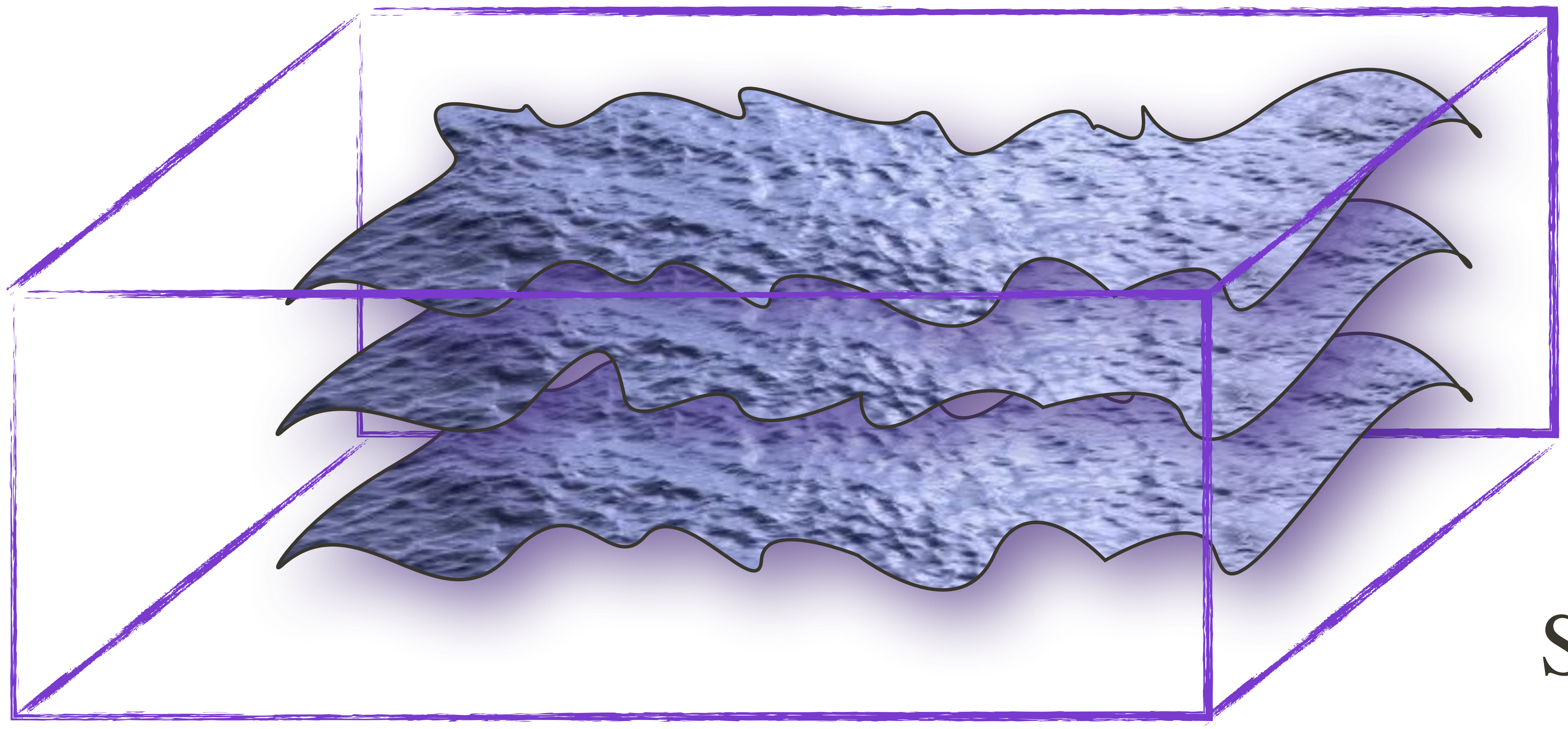
- A concrete realization of a quantum theory of the gravitational field ✓
- That is well defined without uncontrollable infinities ✓
- Whose classical limit is General Relativity ✓
- In 4 Lorentzian dimensions ✓
- With the standard matter couplings ✓


Dispensable!
Background Spacetime
(Continuos) Manifold
Dispensable!
(Generalized) Phase Space
Quantum Theory
Symplectic Geometry
General Covariance
Discreteness
Relational Evolution
Partial Observables
Gauge Theories
Gravitational Field
Reference Fields

Minimalism is revolutionary!

EINSTEIN 1915

- General Relativity: spacetime is the gravitational field (substance)
- background independence / general covariance

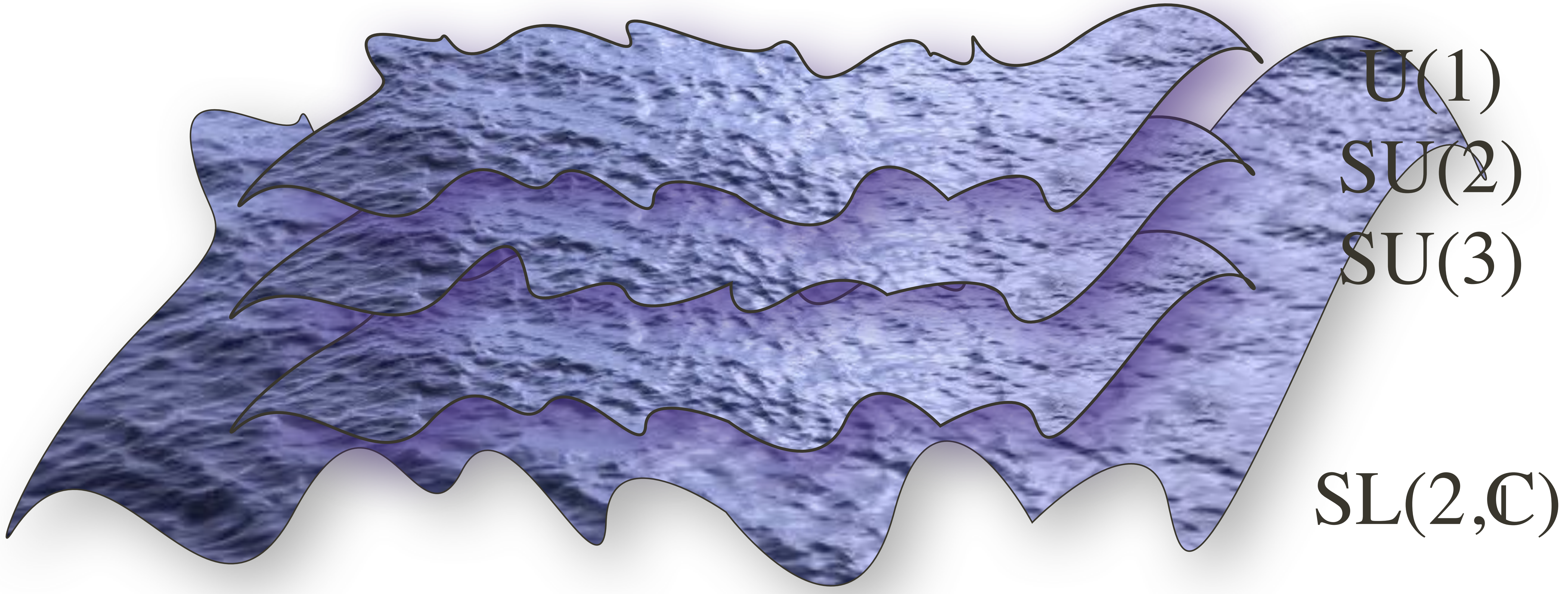


FIELDS ⟷ GAUGE SYMMETRIES

Gravity as an interacting gauge field

EINSTEIN 1915

- General Relativity: spacetime is the gravitational field (substance)
- background independence / general covariance



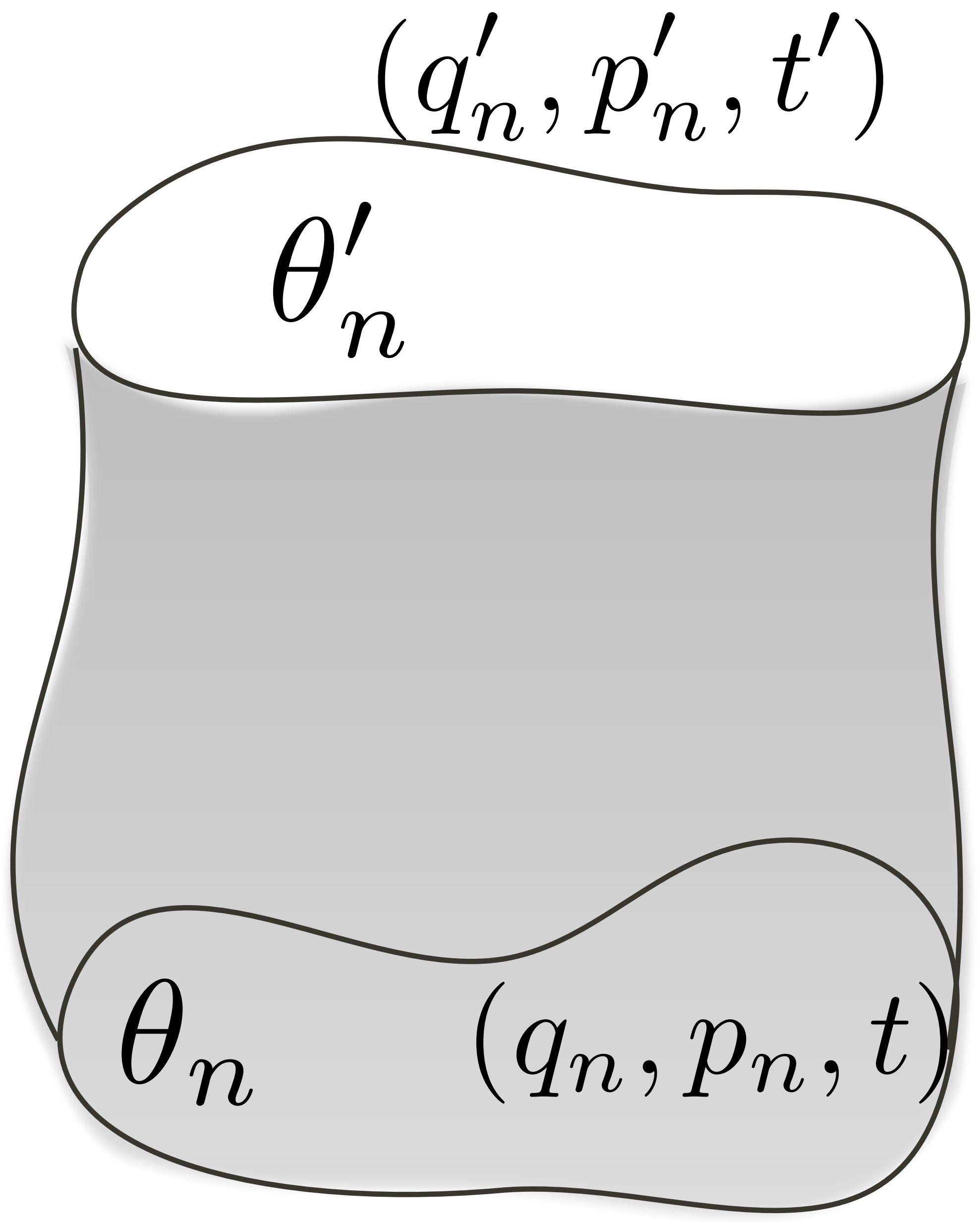
FIELDS ⟷ GAUGE SYMMETRIES

Gravity as an interacting gauge field

BOUNDARY FORMALISM

(Oeckl 2003)

- Democratization of gauges: they all determine dynamical constraints among partial observables measured at the boundaries of a process
- The full content of a dynamical theory is in the constraints!
  - Yang-Mills constraint determines variable change wrt a change of the internal boundary frame.
  - Diff constraint determines variable change wrt a change in the location of the spatial boundary reference frame.
  - Hamiltonian constraint determines collective variable change wrt a change in the temporal location of the boundary (time of measurement)
- Time is pure gauge, the Hamiltonian constraint determine time evolution
- Indeterminacy ⟷ arbitrariness of the frame choice
- Dynamics is the study of relations between partial observables that are gauge-dependent quantities of a system to which we can couple an apparatus

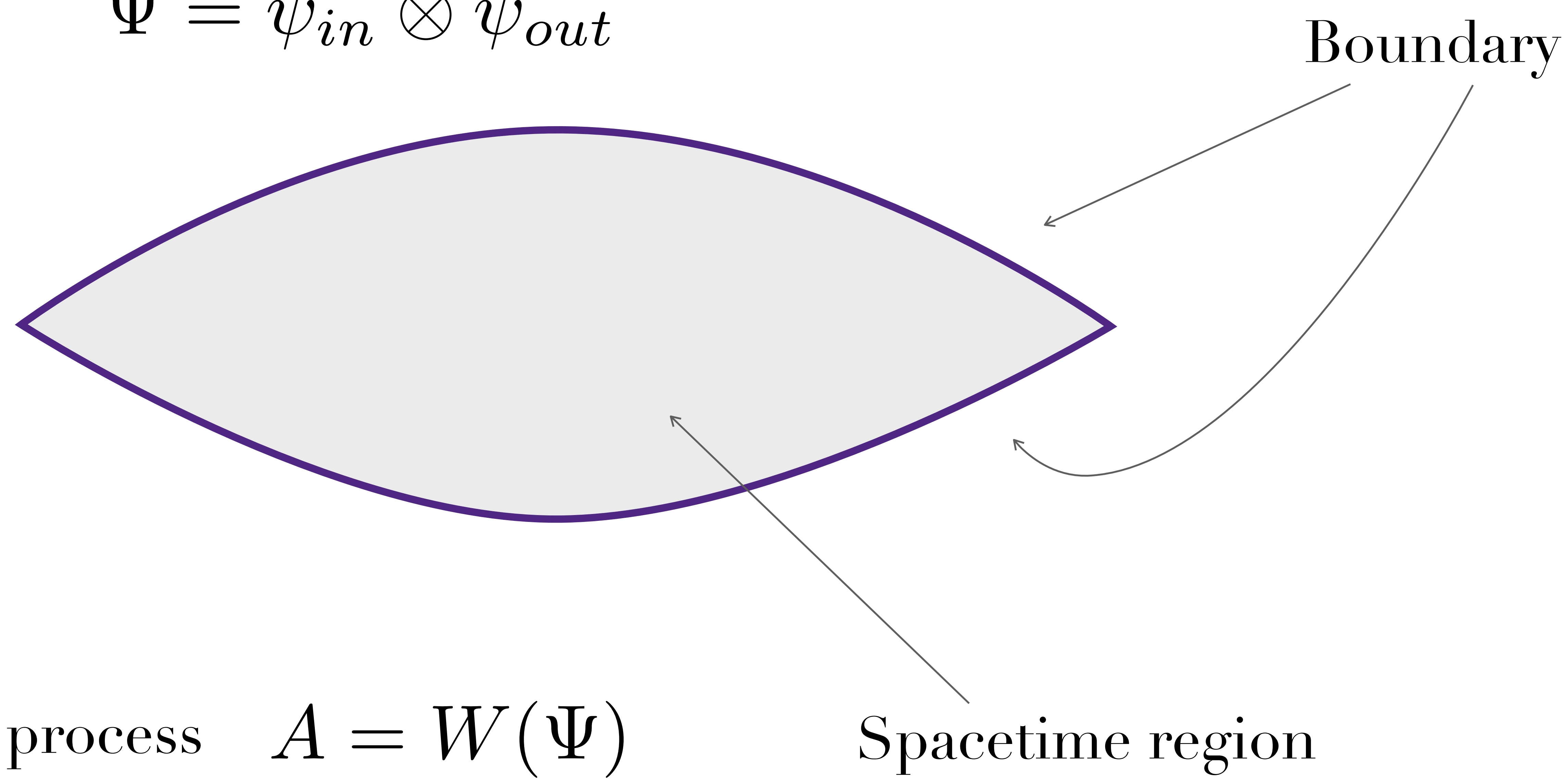


SPACETIME IS A PROCESS

QUANTUM MECHANICS		GENERAL RELATIVITY
Process	← Locality →	Spacetime region
State		Boundary, space region

Spacetime is a process, a state is what happens at its boundary.

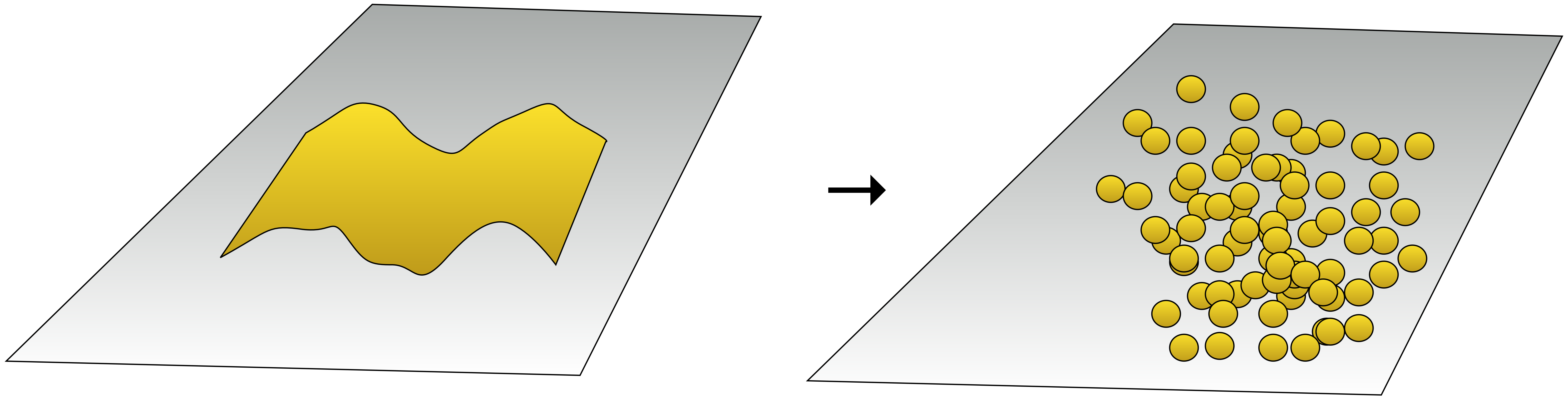
Boundary state $\Psi = \psi_{in} \otimes \psi_{out}$



Amplitude of the process $A = W(\Psi)$


QUANTUM DISCRETENESS

PRELIMINARY: RECALL Q.E.D.

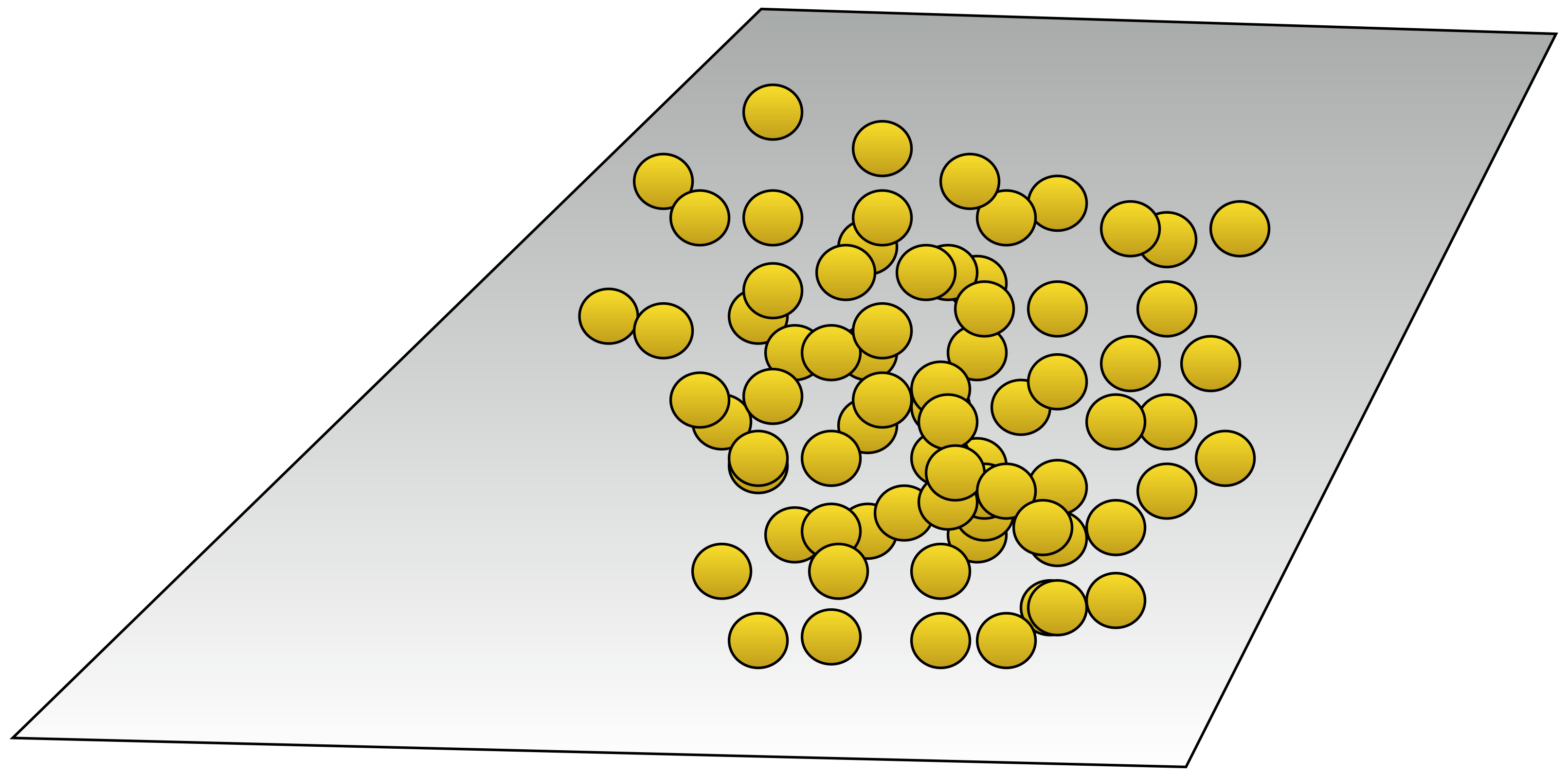


The quanta of a field are particles (Dirac).
DISCRETENESS of the spectrum of the energy of each mode

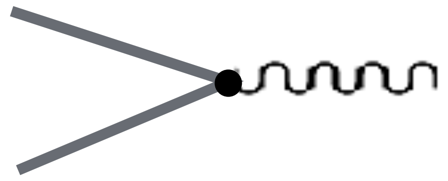
$$(\mathcal{F}, \mathcal{A}, W)$$

- STATES $\mathcal{F} \ni |p_1 ... p_n\rangle$
- OBSERVABLES $\mathcal{A} \ni a(k), a^\dagger(k)$
- DYNAMICS $W \to Feynman\ rules$

LIMIT $\hbar \longrightarrow 0$



- DISCRETE
- FUZZY
- PROBABILISTIC



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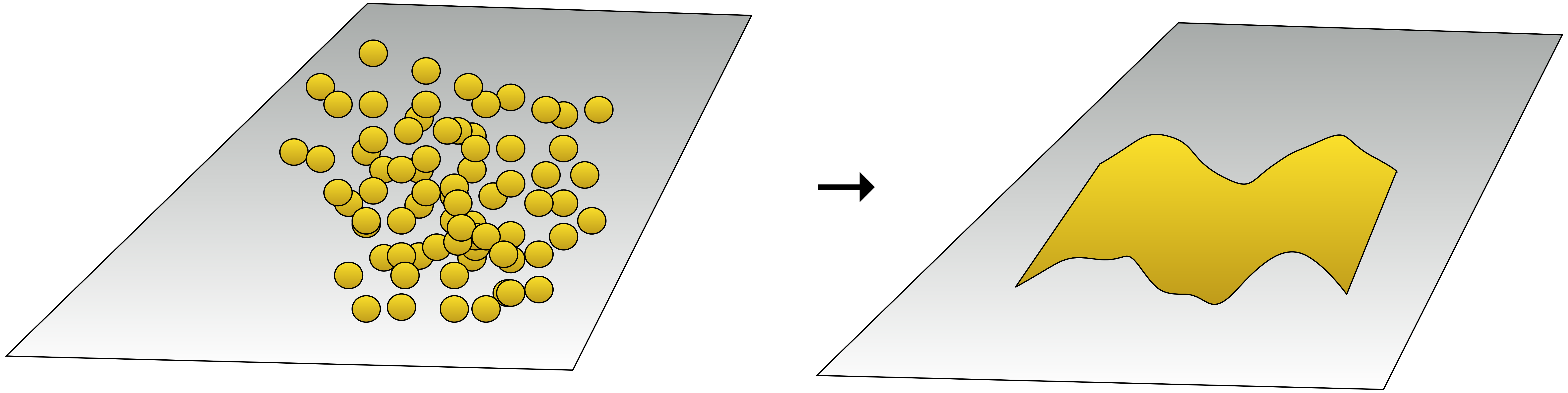

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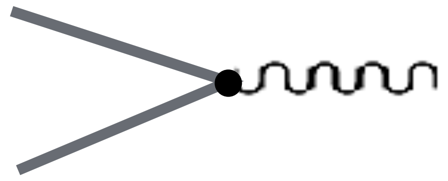
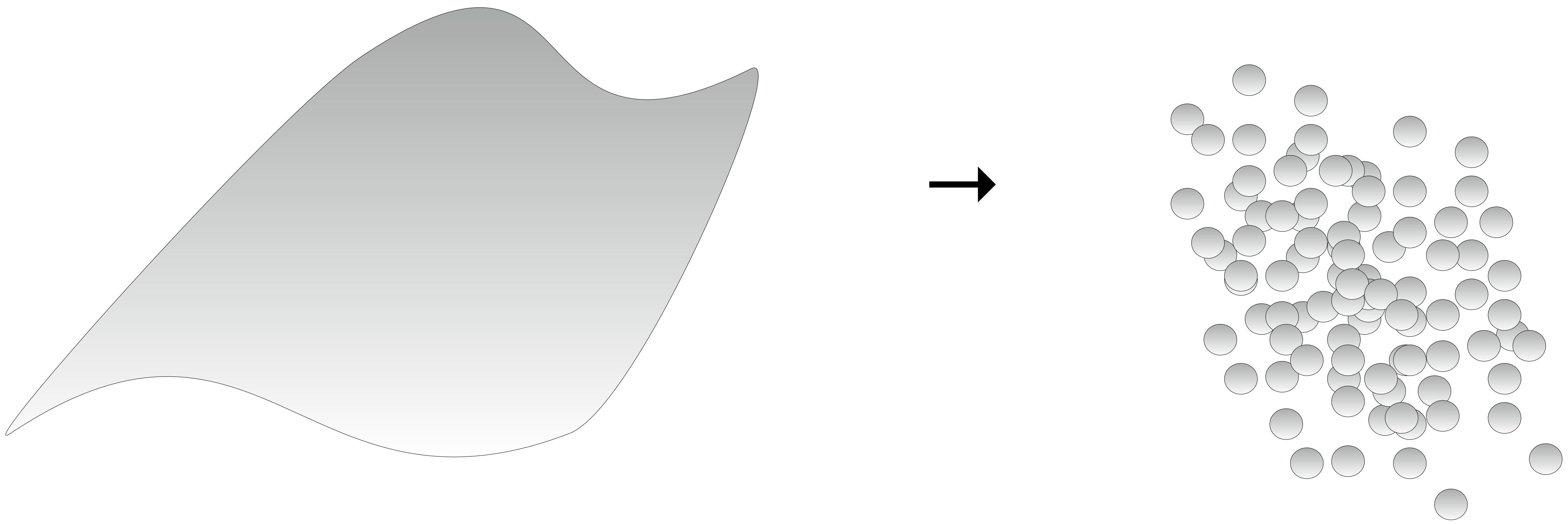


- NO DISCRETENESS
- NO FUZZYNESS $A_\mu(x) = \int d^3x \, (a(x)e^{+ikx} a^\dagger(x) e^{-ikx})$
- A CLASSICAL FIELD $A_\mu(x)$

QUANTUM GRAVITY



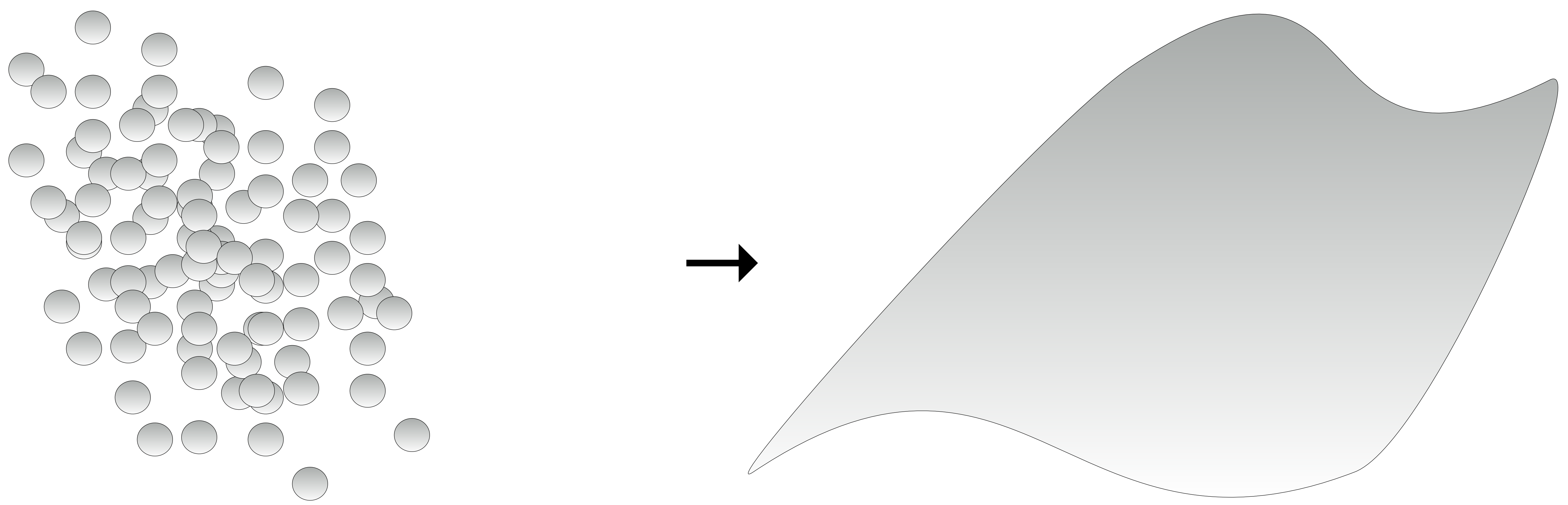
Quantum granularity of spacetime (Rovelli&Smolin 1995)

DISCRETENESS of the spectrum of geometrical operators such as volume

$(\mathcal{H}, \mathcal{A}, \mathcal{W})$

- STATES $\mathcal{H} \ni |\Gamma, j_l, v_n\rangle$
- OBSERVABLES $\mathcal{A} \ni \vec{L}_l$
- DYNAMICS $W \rightarrow Transition\ amplitudes$

LIMIT $\hbar \longrightarrow 0$



- DISCRETE $\ell_{Pl}^2 = \hbar G$
- FUZZY
- PROBABILISTIC

- NO DISCRETENESS $\ell_{Pl} \to 0$
- NO FUZZYNESS
- A CLASSICAL FIELD $E_a^i(x) \to g_{\mu\nu}(x)$

FROM QUANTUM TO CLASSICAL

- ## FROM QUANTUM TO CLASSICAL

  The *classical* limit is $\hbar \longrightarrow 0$, the limit for ∞ quanta is relevant for the *continuous* limit

  No thermodynamical limit is needed at that stage.

- ## EMERGENCE OF SPACETIME IS STANDARD CLASSICAL EMERGENCE

  just as the electromagnetic field emerges from photons

- ## SPACETIME IN THE QUANTUM REGIME IS MADE OF QUANTA

  - there is no classical spacetime in the quantum regime
  - same as in Q.E.D. where there are photons

- ## SPACETIME IN THE QUANTUM REGIME IS A QUANTUM PROCESS

  - states are defined by the continuity relations between quanta
  - a spinfoam is a quantum interaction, but also a spacetime region

- ## THERE IS NO TIME, THERE IS ONLY CHANGE

  in fact change is everything we measure!

QUANTUM GRAVITY IS THE DISCOVER OF A MINIMAL LENGHT

- QUANTUM MECHANICS

Heisenberg Uncertainty $\Delta x > \hbar / \Delta p$

Sharp localization requires large energy. $E \sim cp$

- GENERAL RELATIVITY $M \sim E/c^2$

Black-Hole Horizon $R \sim GM/c^2$

The horizon prevent a sharper localization. $\Delta x \geq R$

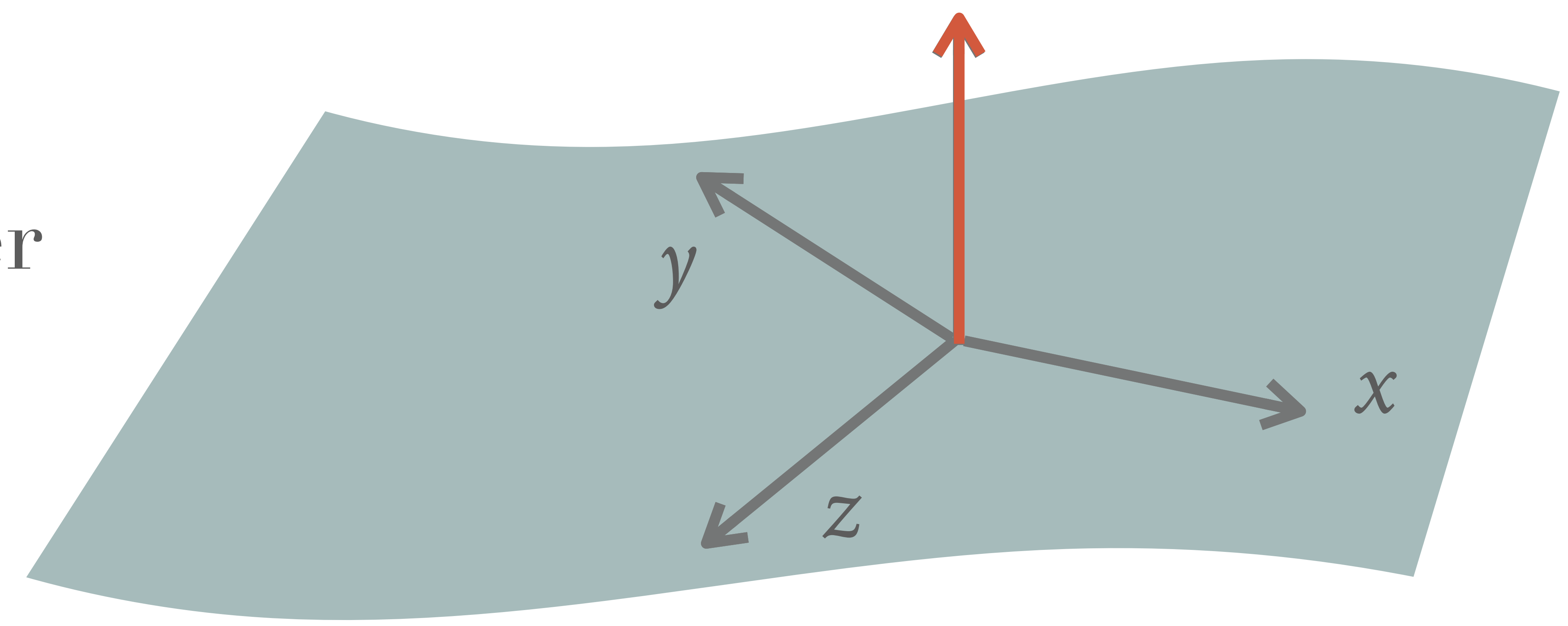
- QUANTUM GRAVITY

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}\, m$$

"Without a deep revision of classical notions it seems hardly possible to extend the quantum theory of gravity also to [the short-distance] domain."

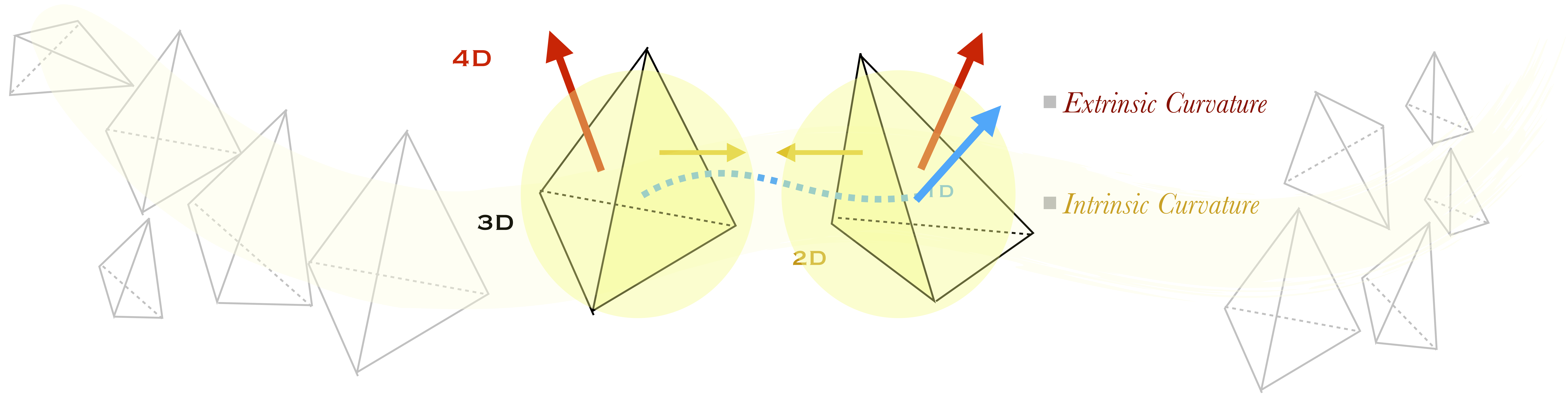
Matvei Bronstein

GENERAL RELATIVITY



- Tetrads (reference fields): required for coupling matter
- ADM formalism: select a foliation at a given time
- Hamiltonian formulation: (densitized) triads are conjugate to the Ashtekar connection
- Triads are rotation invariant: $so(3) \longrightarrow su(2)$

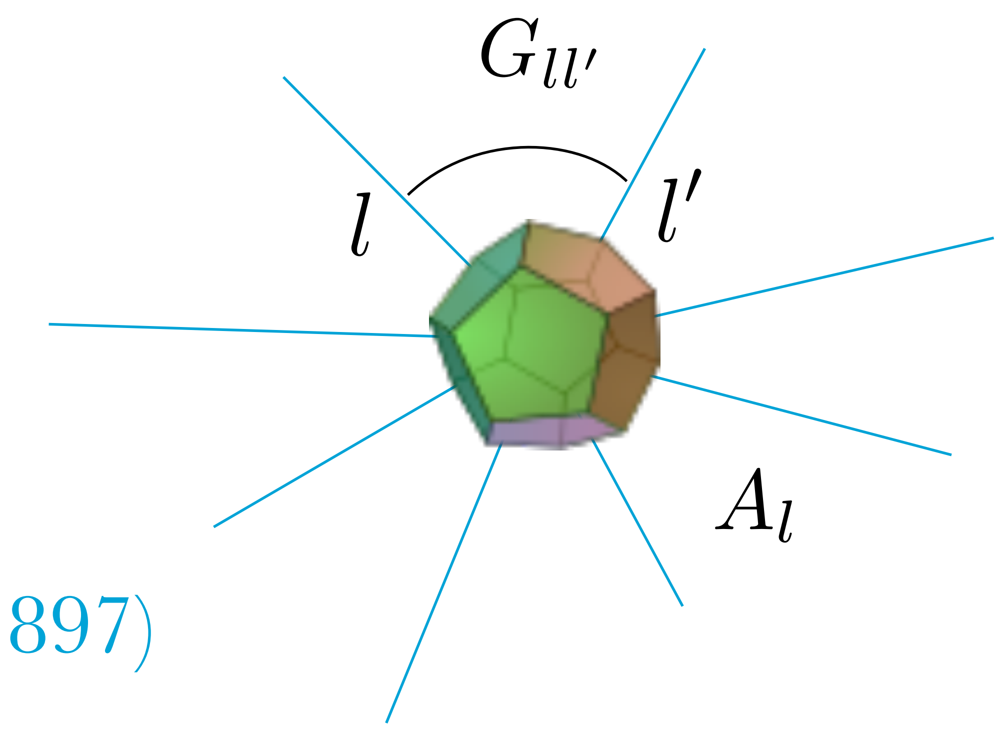
QUANTUM GEOMETRY



- h_l "Holonomy of the Ashtekar-Barbero connection along the link"
- $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ SU(2) generators $\quad L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$

  gravitational field operator (tetrad)

  - Gauge invariant operator $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ with $\sum_{l\in n} G_{ll'} = 0$



Penrose's **spin-geometry theorem** (1971), and **Minkowski theorem** (1897)

REPRESENTING GEMETRIES

- Composite operators:
  - Area: $$A_\Sigma = \sum_{l\in\Sigma} \sqrt{L_l^i L_l^i}.$$
  - Volume: $$V_R = \sum_{n\in R} V_n, \qquad V_n^2 = \frac{2}{9}\,|\epsilon_{ijk} L_l^i L_{l'}^j L_{l"}^k|.$$
  - Angle: $$L_l^i L_{l'}^i$$

- Geometry is quantized:
  - eigenvalues are discrete
  - the operators do not commute
  - quantum superposition

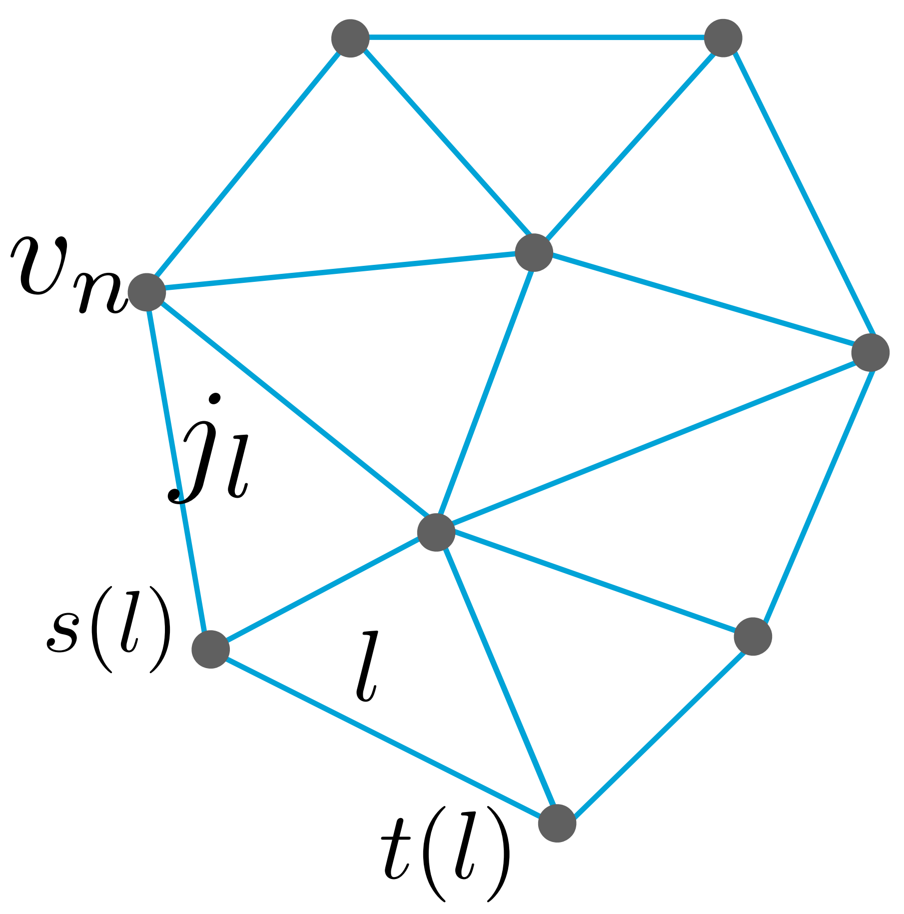
    ↳ *coherent states*

Quantum states of space,
rather than states on space.

GRAVITY AS A GAUGE THEORY

GAUGES IN GENERAL RELATIVITY $\longrightarrow$ $\begin{cases} \text{- diffeos} \\ \text{- Lorentz} \end{cases}$ $\longrightarrow$ $\begin{cases} \text{- graph/lattice} \\ \text{- group variables} \end{cases}$ $\longrightarrow$ LOOP QUANTUM GRAVITY

- Abstract graphs: $\Gamma = N, L$
- Group variables: $\begin{cases} h_l \in SU(2) \\ \vec{L}_l \in su(2) \end{cases}$
- Graph Hilbert space: $\mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N]$
- The space $\mathcal{H}_\Gamma$ admits a basis $|\Gamma, j_\ell, \nu_n >$



- MANIFOLD is introduced as a device to connect with the classical limit

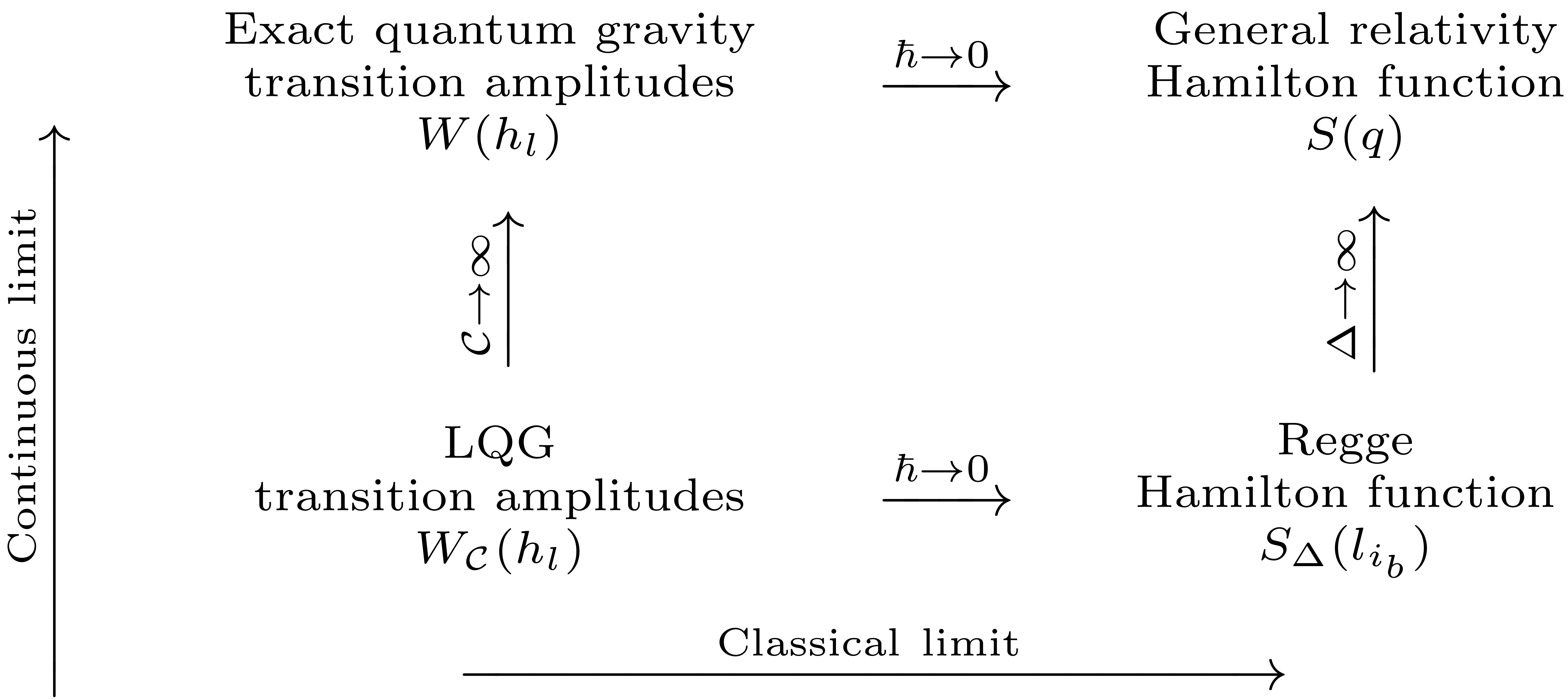
LOCALIZATION

- COUPLING TO A MATERIAL REFERENCE SYSTEM
  The gravitational-field components with respect to the directions defined by the matter system are gauge-invariant quantities of the coupled system; but they are gauge-dependent quantities of the gravitational field, measured with respect to a given external frame.

- MEASUREMENT VS PREDICTIONS
  Gauge variables are what we measure but we cannot predict.
  A couple of gauge variables is gauge invariant and it allows predictions.

- RELATIONALITY
  Bodies are only localized with respect to one another.
  Bodies includes all dynamical objects, also the gravitational field.
  Spacetime is built up by contiguity relations: being "next to one another".

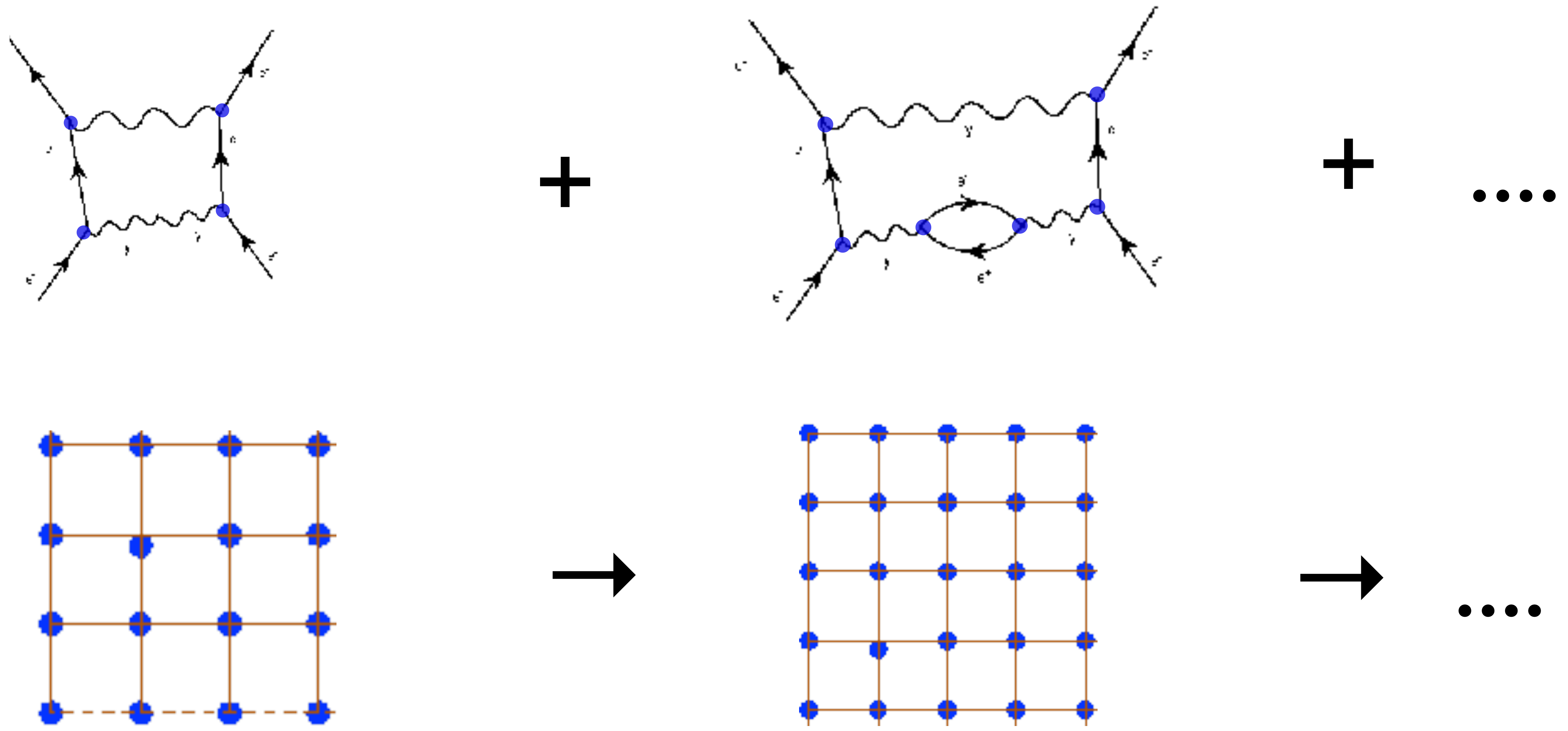
STRUCTURE OF THE THEORY



- No critical point
  - QFT : critical phenomenon
  - Quantum Gravity: non-critical phenomenon
- No infinite renormalization
- Physical Scale ℓ_{Pl}

CONVERGENCE BETWEEN QED AND QCD

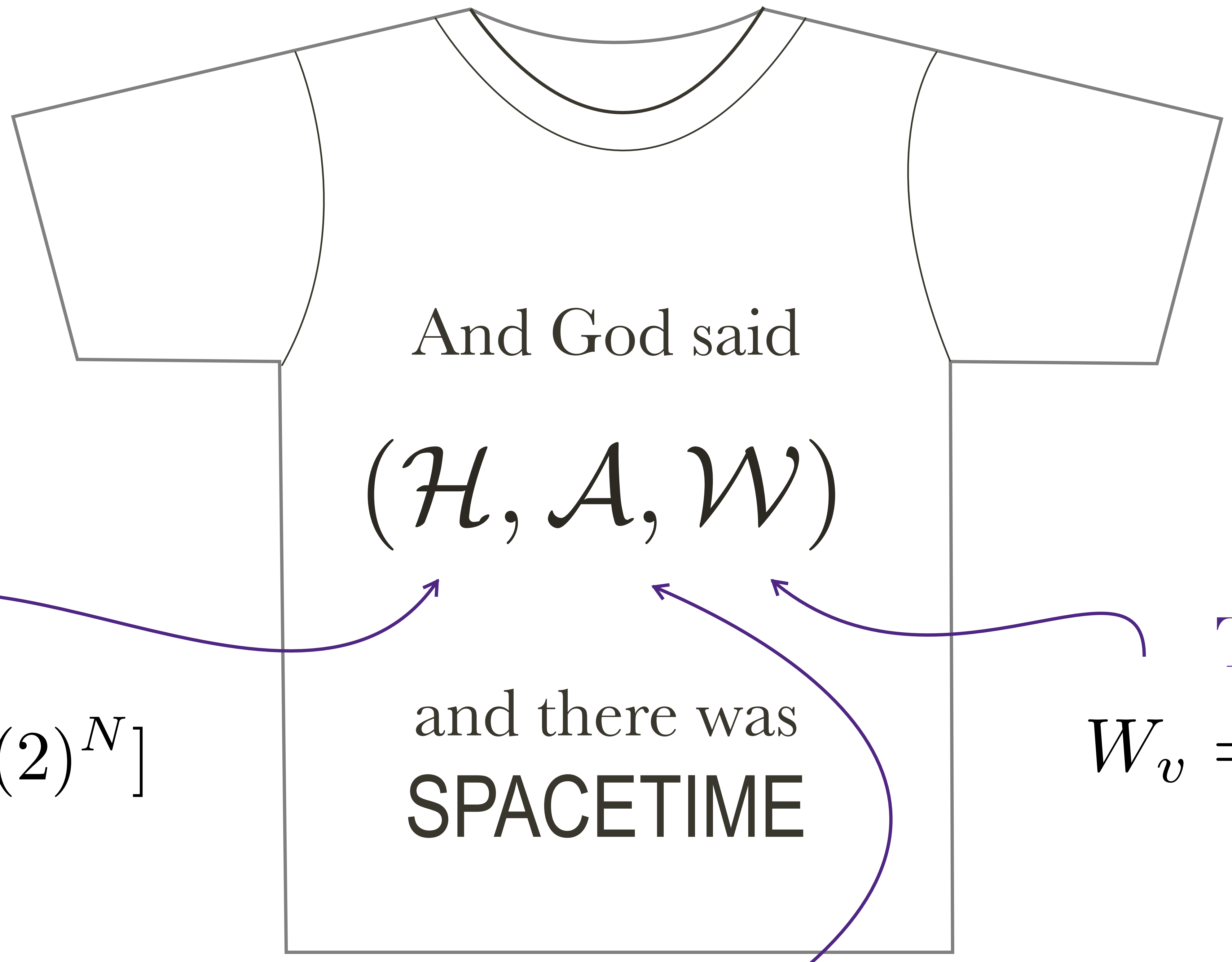
- All physical QFT are constructed via a truncation of the d.o.f. (QED: particles, QCD: lattice)
- All physical calculation are performed within a truncation.
- The limit in which all d.o.f. is then recovered is pretty different in QED qnd QCD:



Matter coupling as in LATTICE GAUGE THEORIES but without divergences!

- Quantum Gravity: Diff invariance !
- Lattice site = small region of space = excitations of the gravitational field = quanta of space = quanta of the field

THE THEORY



Hilbert Space:

$$\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$$

Transition Amplitude:

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbb{1})$$

Operator Algebra:

$$[L_a^i, L_b^j] = i\delta_{ab}\ell^2\epsilon^{ij}_k L_a^k$$

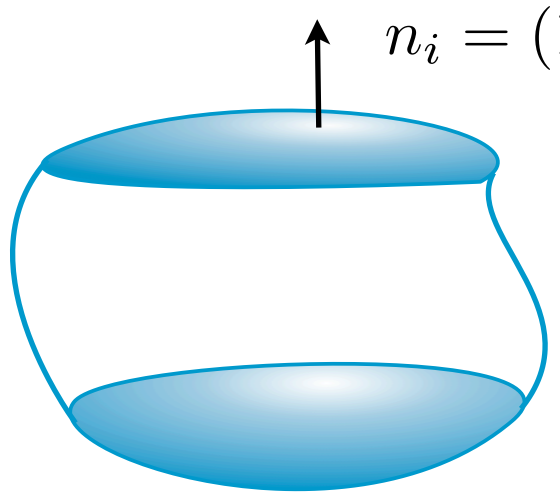
Covariant Loop Quantum Gravity, with C. Rovelli

CLASSICAL THEORY: TETRADS

- Variables $e = e_a dx^a \in \mathbb{R}^{(1,3)}$ and $\omega = \omega_a dx^a \in sl(2,\mathbb{C})$

- Action $S[e,\omega] = \int B[e] \wedge F[\omega]$ where $B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$

- Boundary
gauge s.t. tetrads are diagonal $B^{oi} = K^i = \frac{1}{\gamma}\, e^o \wedge e^i$ and $B^{ij} = L^i = e^o \wedge e^i$



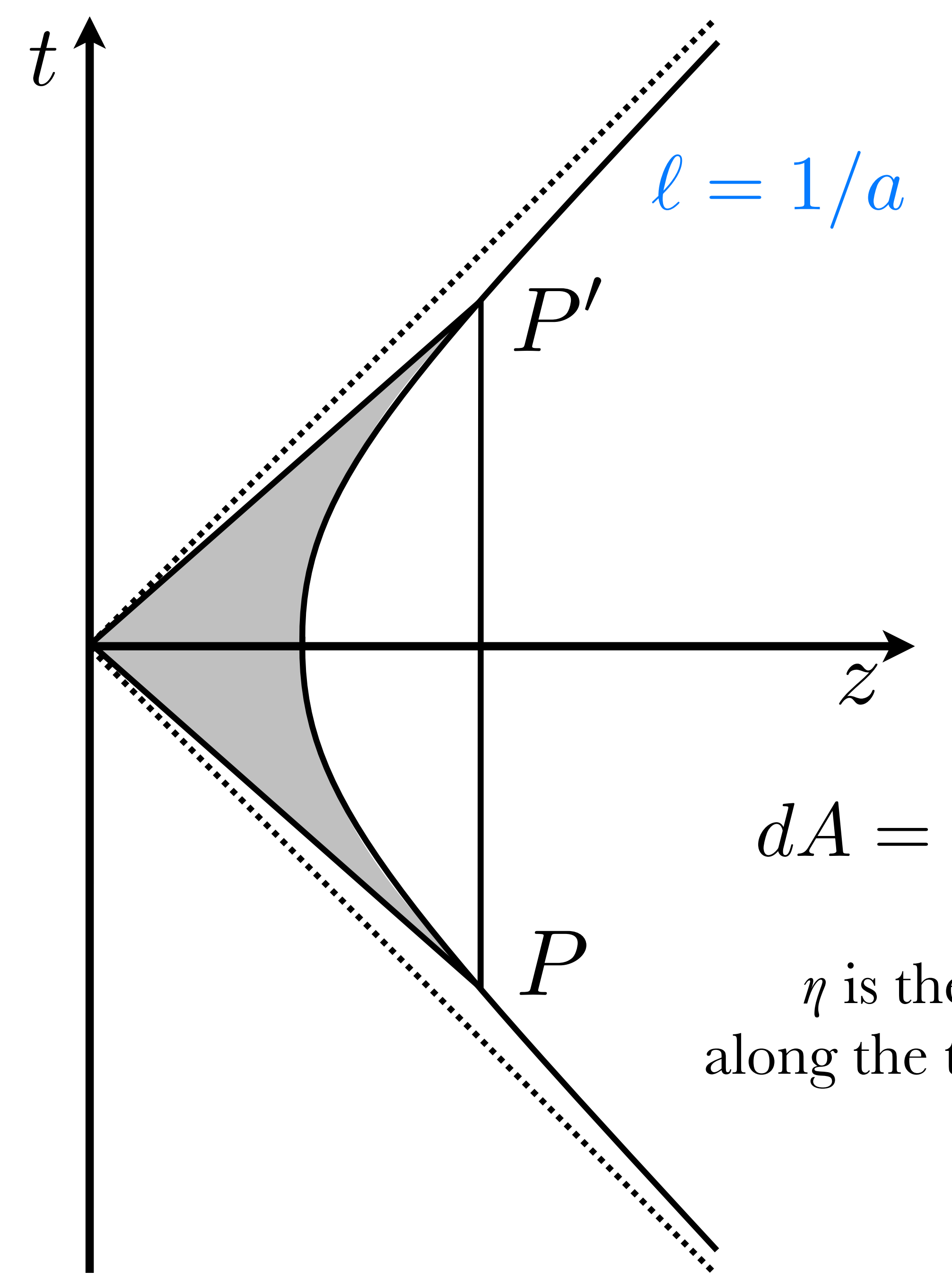
- Simplicity constraint $\vec{K} = \gamma \vec{L}$

$SL(2,\mathbb{C}) \to SU(2)$

- Lorentzian area $A = \int_{\mathcal{R}} e^o \wedge e^i = \int_{\mathcal{R}} \gamma K^i = \int_{\mathcal{R}} L^i$

THERMODYNAMICS

Frodden, Gosh, Perez 1110.4055



$$dA = \frac{\ell^2}{2} d\eta = \frac{1}{2a^2} d\eta$$

η is the boost parameter along the trajectory from P to P′

- Constantly accelerated observer:
  - K generator of boost
  - E=aK generator of proper time evolution

Linear simplicity constraint $B^{0k} = \gamma \epsilon^{k}{}_{ij}\ B^{ij}$

$$\vec{K} = \gamma \vec{L}$$

- Simplicity Constrain

  Physics is defined locally!

$$E = \frac{A}{8\pi G}\, l^{-1} \quad \Rightarrow \quad S_{\mathrm{BH}} = \frac{A}{4\, G\hbar}$$

$$T = \frac{\hbar a}{2\pi}$$

$$H_A = 2\pi \textstyle\sum_l K_l$$

$$\rho_A = e^{-H_A}$$

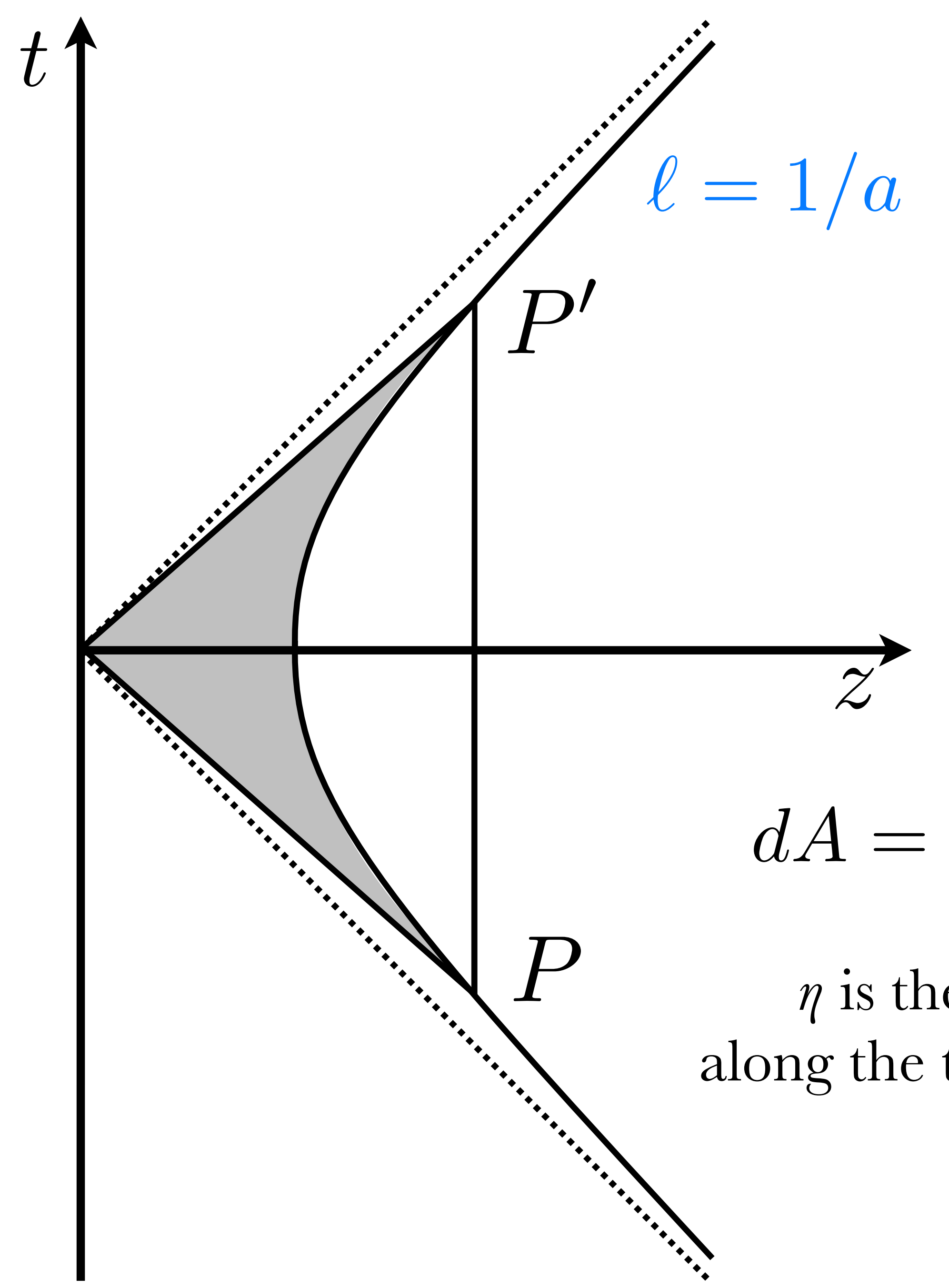
$$S_{\mathrm{EE}} = -\mathrm{Tr}(\rho_A \log \rho_A) = 2\pi \mathrm{Tr}(\textstyle\sum_l K_l\ \rho_A)$$

$\hbar$

$$S_{\mathrm{EE}} = \frac{\mathcal{A}_\Sigma}{4G_0}\ .$$

Bianchi 1204.5122

MAXIMAL ACCELERATION



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Linear simplicity constraint $B^{0k} = \gamma \epsilon^k{}_{ij} \, B^{ij}$

$$\vec{K} = \gamma \vec{L}$$

- Lorentzian area: $A = \int_{\mathcal{R}} \gamma K^z = \int_{\mathcal{R}} L^z$

$$A_{min} = 4\pi G\hbar \qquad a_{max} = \sqrt{\frac{1}{8\pi G\hbar}} \qquad \ell_{min} = \sqrt{8\pi G\hbar}$$

[Cainiello ’81]
[Cainiello, Gasperini, Scarpetta ’91]
[Bozza, Feoli, Lambiase, Papini, Scarpetta]

NO CURVATURE SINGULARITIES IN LQG

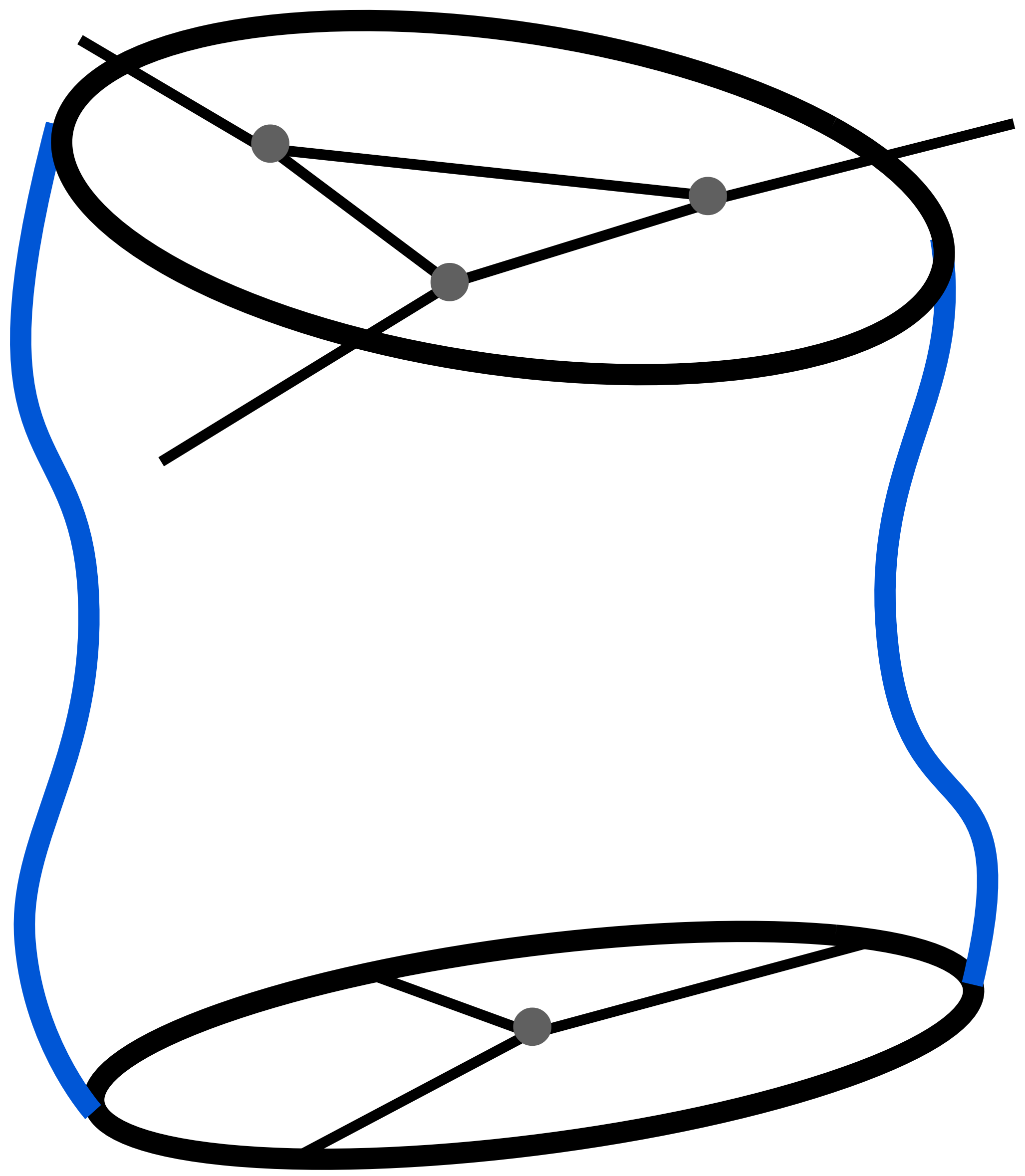
TO CONCLUDE

SPINFOAM AMPLITUDES

Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

for a state Ψ associated to the boundary of a 4d region

$$W(q'_{ij}, q_{ij}) \sim \int_{\partial g = q', q} Dq \; e^{i\,S}$$



- Superposition principle $\langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Locality: vertex amplitude $W(\sigma) \sim \prod_v W_v.$
- Lorentz covariance $W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbb{1})$
- UV and IR finite (with Λ)
- Classical limit: GR (with Λ)
  (via Regge discretization)

Barrett et al. '09

www.cpt.univ-mrs.fr/~rovelli/IntroductionLQG.pdf

The path to

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IN THIS LECTURE

- THEORY: Loop Quantum Gravity — *Covariant Loop Quantum Gravity, with C. Rovelli*
- SETTING: Primordial Universe — *Relational Quantum Cosmology*
- APPLICATION: Quantum State of the Primordial Universe — *Primordial fluctuations from quantum gravity, with F. Gozzini*

APPLICATION TO COSMOLOGY

- **Quantum Regime**
- Graph Approximation
- Hartle-Hawking State
- Transition Amplitude
- Computational Method

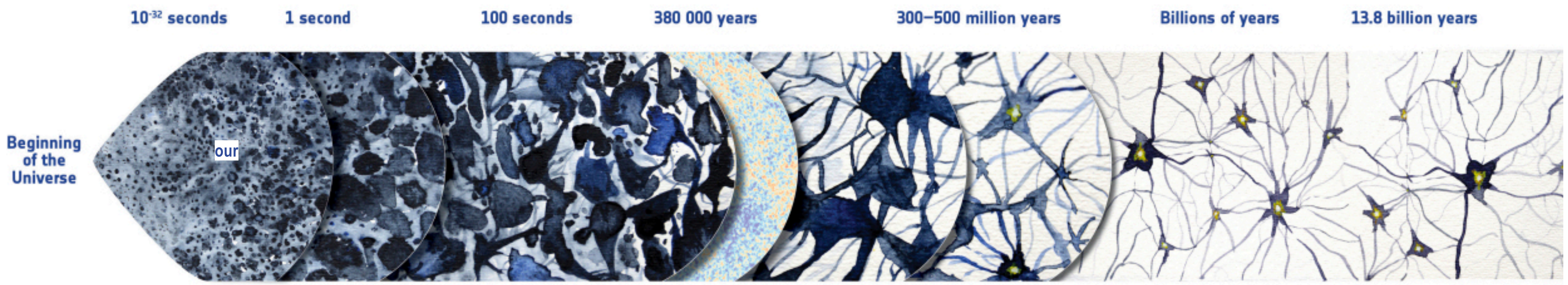

Image credit: ESA

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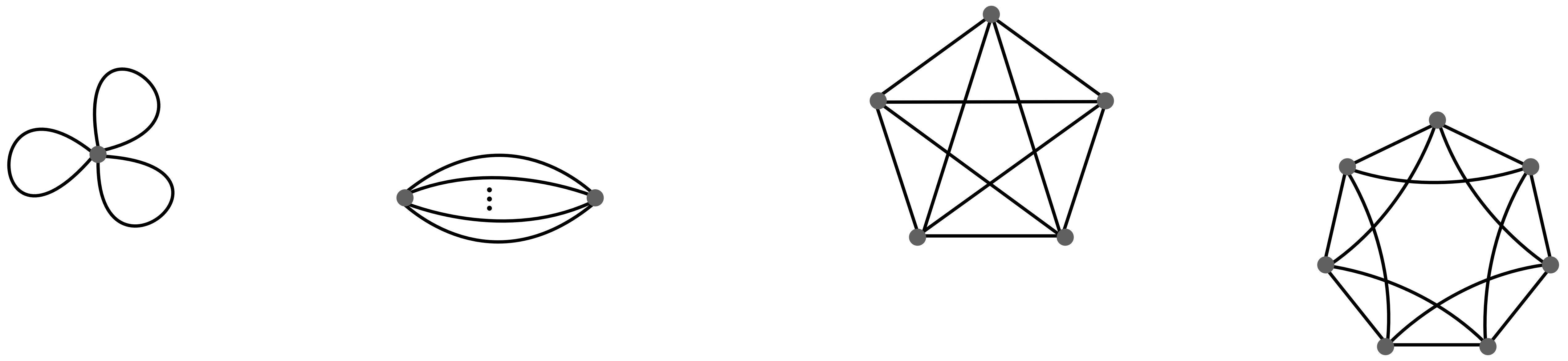

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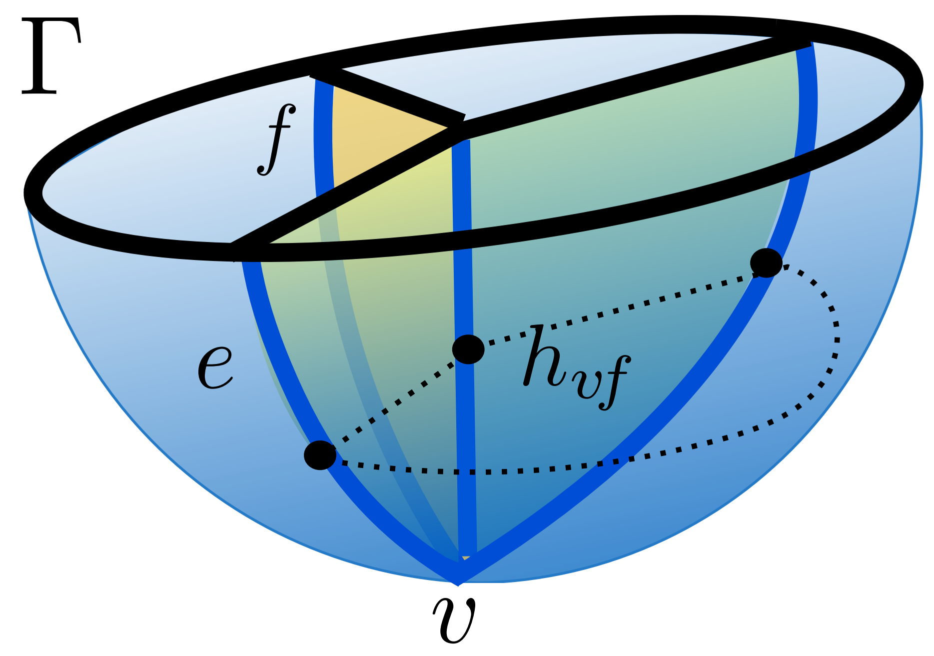
Spinfoam Hartle-Hawking state



Toward Spinfoam Cosmology, with E. Bianchi and C. Rovelli

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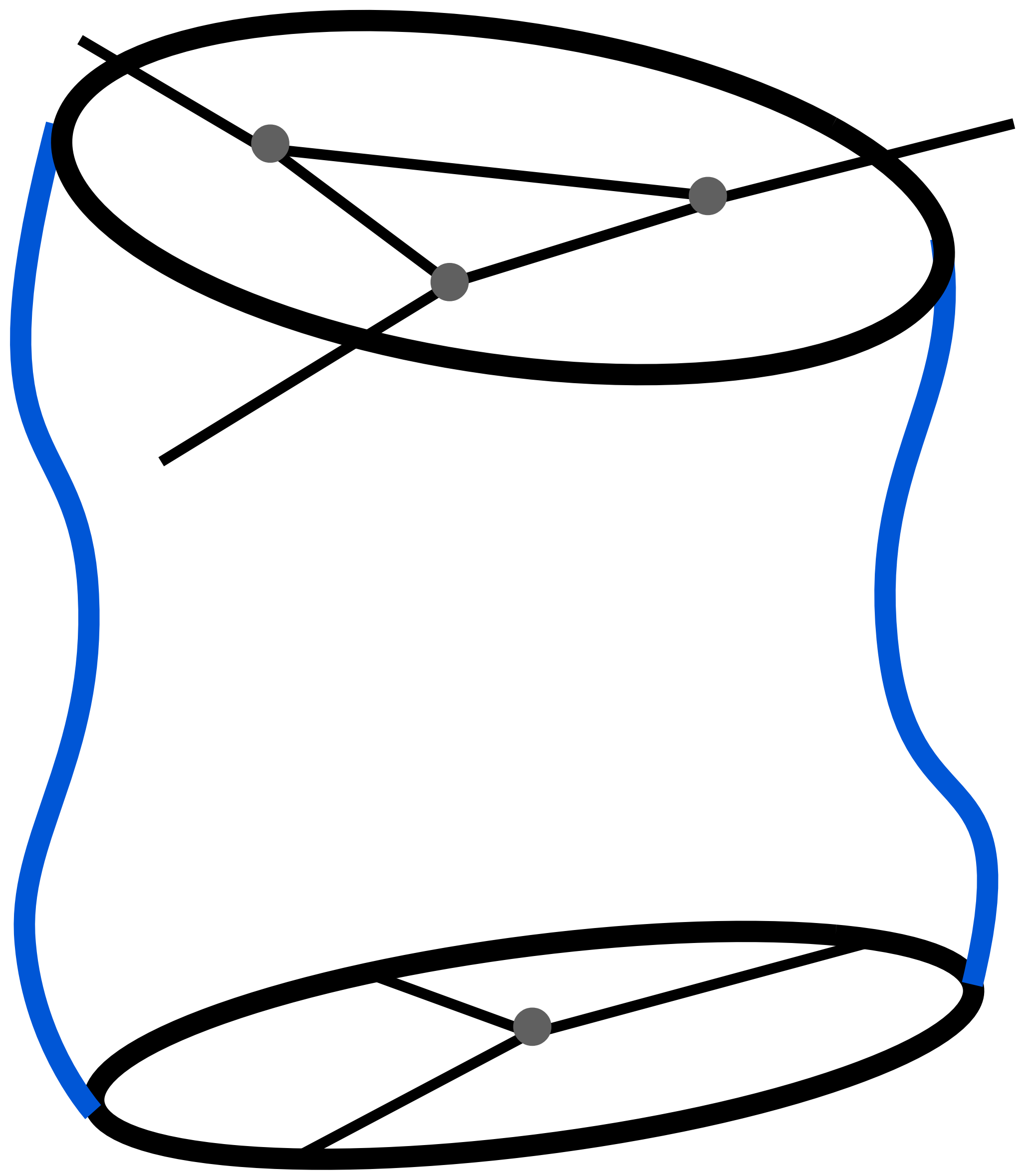

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APPLICATION TO COSMOLOGY

- Quantum Regime
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- Transition Amplitude
- **Computational Method**

$$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

1^{ST}-ORDER FACTORIZATION

- classical dynamics $H = const\,(a\dot{a}^2 - \frac{\Lambda}{3}a^3) = 0$ $\dot{a} = \pm\sqrt{\frac{\Lambda}{3}}a$

1ST-ORDER FACTORIZATION

- classical dynamics $S_H = const \int \mathrm{dt}\,(a\dot{a}^2 + \frac{\Lambda}{3}a^3)\Big|_{\dot{a}=\pm\sqrt{\frac{\Lambda}{3}}a} = const\frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a_f^3 - a_i^3)$
- quantum dynamics $W(a_f, a_i) = e^{\frac{i}{\hbar}S_H(a_f,a_i)} = W(a_f)\overline{W(a_i)}$
- loop dynamics $\langle W|\psi_{H_{(z,z')}}\rangle = W(z,z') = W(z)\,\overline{W(z')}$

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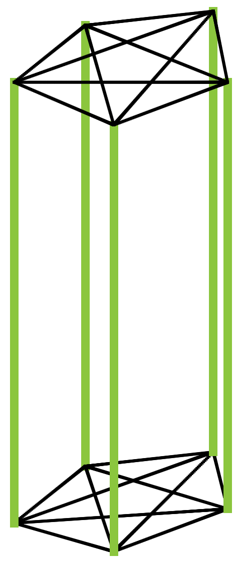
order (0)



$= W_0(h_\ell, h_{\ell'}) = \delta_{\Gamma_\ell}(h_\ell, h_{\ell'})$

1^{ST}-ORDER FACTORIZATION

- classical dynamics $S_H = const \int \mathrm{dt}\,(a\dot{a}^2 + \frac{\Lambda}{3}a^3)\Big|_{\dot{a}=\pm\sqrt{\frac{\Lambda}{3}}a} = const\frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a_f^3 - a_i^3)$
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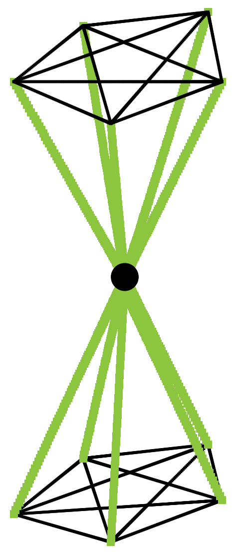
order (1)



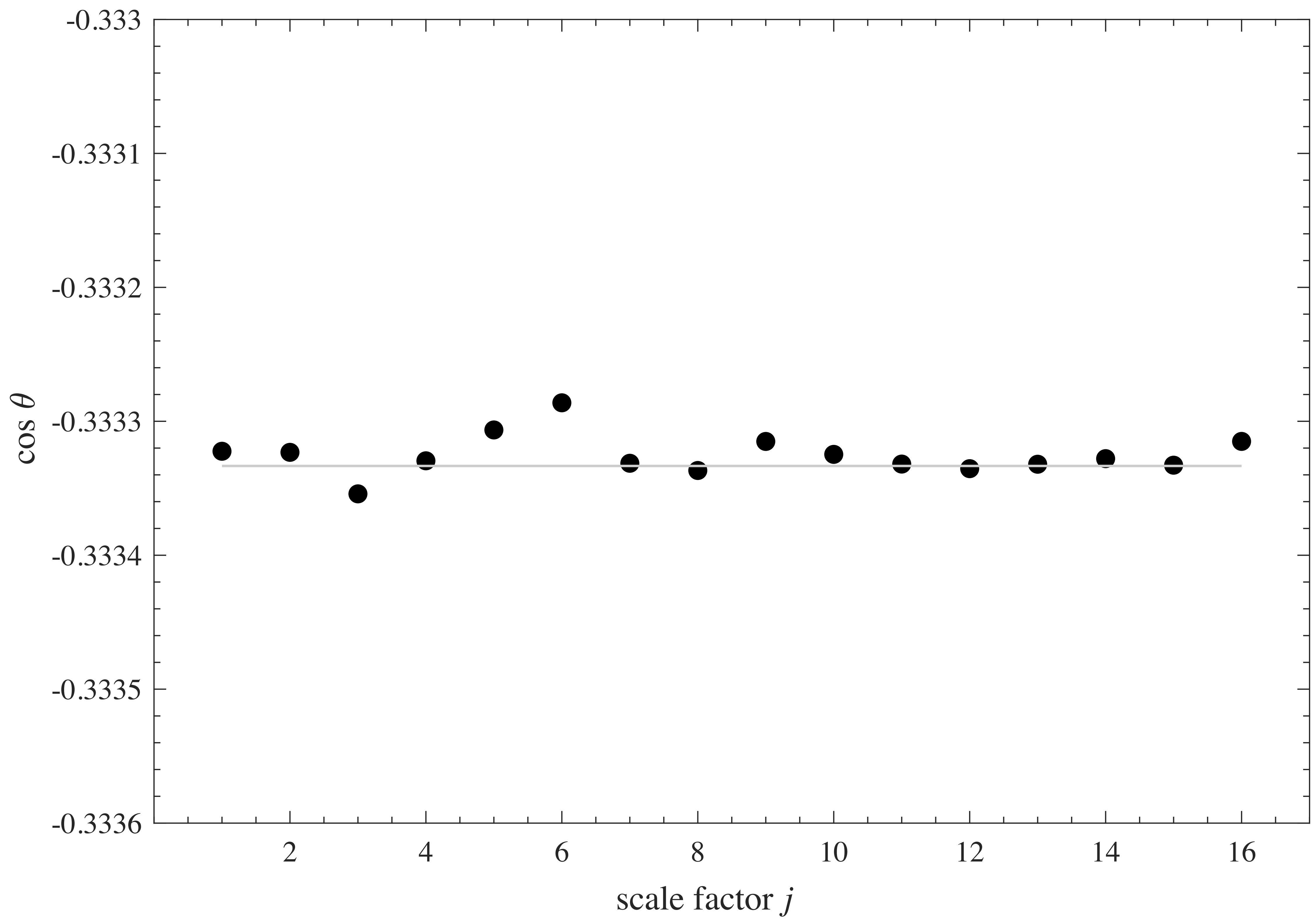
$$W_{\mathcal{C}_\infty}(z',z) = \int h_\ell \int h'_\ell \;\overline{\psi_{z'}(h'_\ell)}\; W_1(h'_\ell,h_\ell)\;\psi_z(h'_\ell)$$

$$W_1(h'_\ell, h_\ell) = \int_{SL(2,\mathbb{C})} \prod_{n=1}^{N-1} dG_n \prod_{\ell=1}^{L} P(h_\ell, G_\ell)P(h'_\ell, G'_\ell)$$

$$G_\ell = G_{n_s}G_{n_t}^{-1}$$

RECENT RESULTS

1. 3-sphere as emerging geometry
2. large fluctuations
3. large correlations



RECENT RESULTS

1. 3-sphere as emerging geometry

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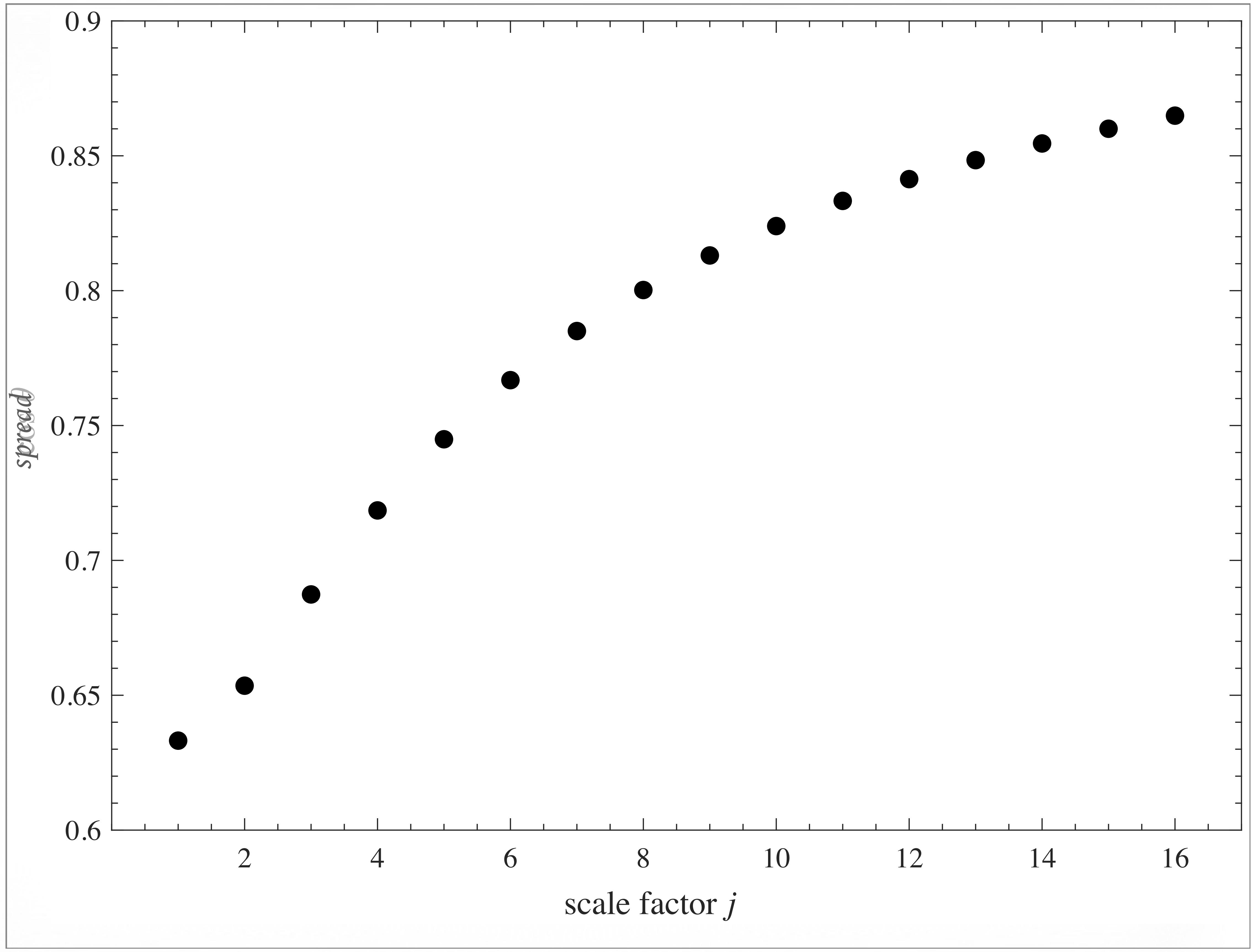

Primordial fluctuations from quantum gravity, with F. Gozzini

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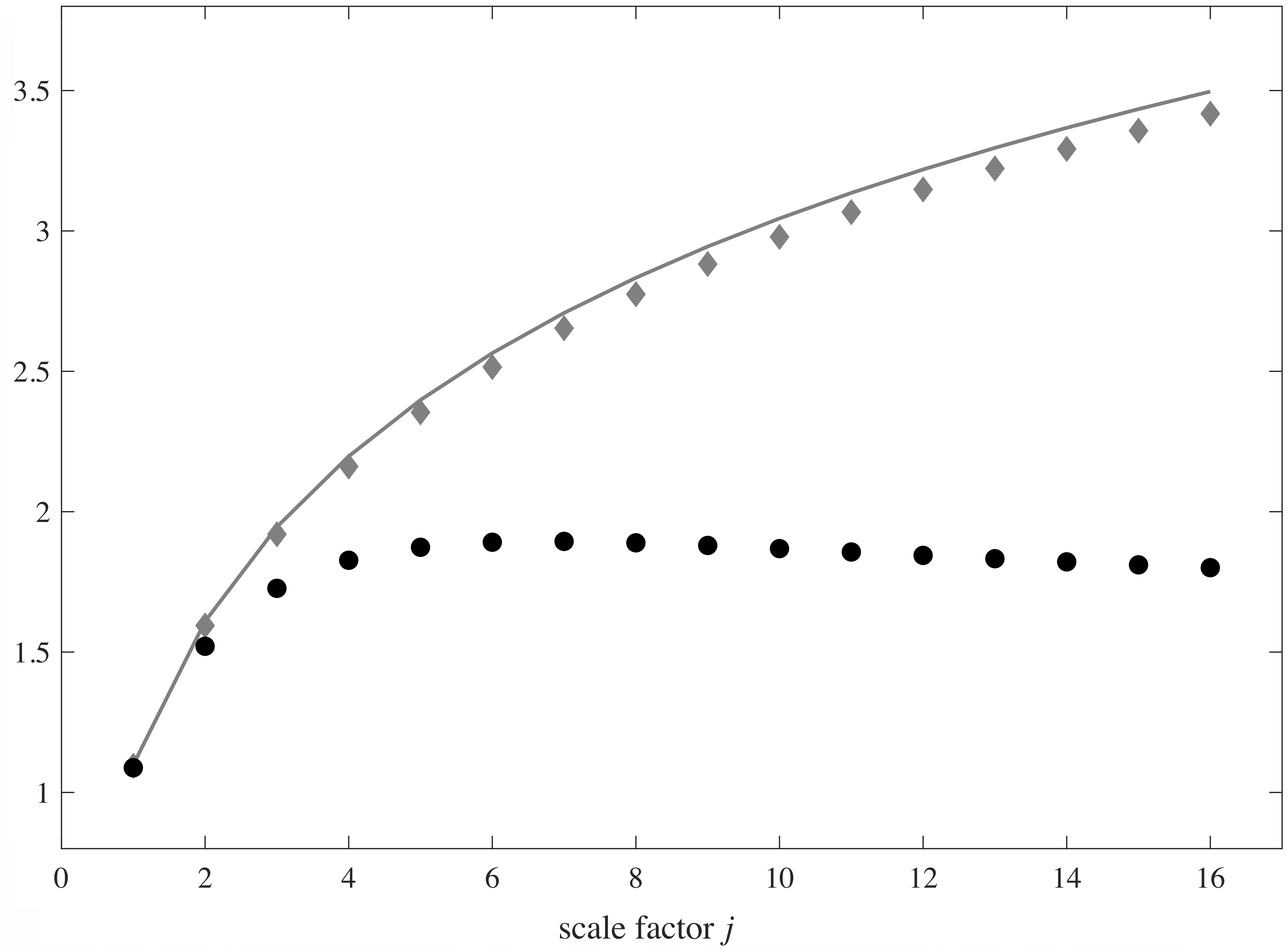

Primordial fluctuations from quantum gravity, with F. Gozzini

CONCEPTUAL QUESTIONS

OPEN QUESTIONS IN QUANTUM GRAVITY

- What are the observables, when localization does not rely on background space and time?
- How should be think about time in this picture?
- What are the degrees of freedom?
- What's the interplay between the Planck scale and the cosmological scale?
- What is the role of quantum fluctuations of spacetime?

1. INTERPRETATION OF THE QUANTUM THEORY

- Computing in quantum mechanics requires a PARTITION

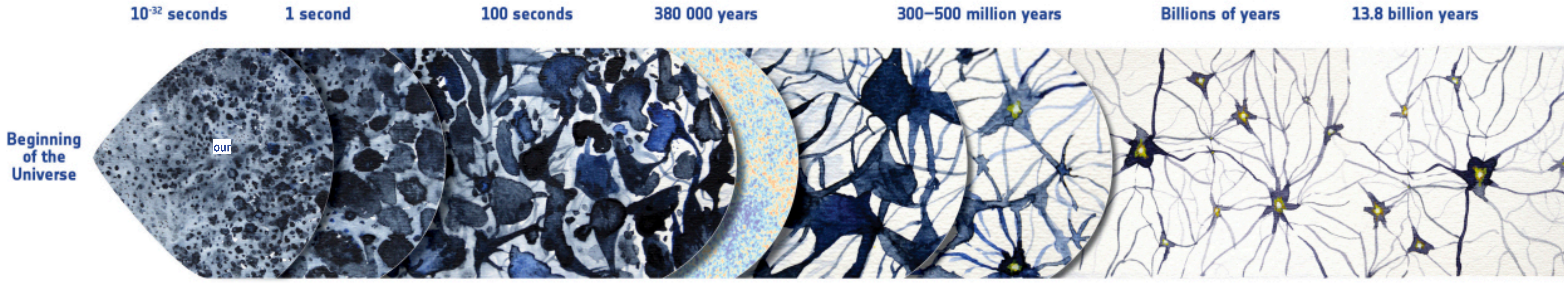

Image credit: ESA

2. WHERE IS SPACE?

1. "*Space*" in the most weak sense: the relation determined by the continuity of things

2. "*Space*" in the sense of geometry: quantity that we can measure with rods

3. "*Space*" in the sense of a continuous Riemaniann space: not in the theory

4. "*Space*" in the sense of the container within which things happen: not in the theory

3. WHERE IS TIME?

1. "*Time*" as change: variables change one with respect to another

2. "*Time*" as preferred time variable: not in the theory

3. "*Time*" in the sense of oriented flowing: compatible but not part of the theory

4. WHAT ARE THE OBSERVABLES?

- In the considered application: area and volumes of space regions
- LOCALIZATION: well defined relationally
    - NO need of continuous space
    - NO need of background space

5. WHAT ARE THE DEGREES OF FREEDOM?

- The same as in General Relativity!
  - quantum numbers j's and v's $\longrightarrow$ $g_{ab}(x)$
- CLASSICAL LIMIT in Quantum Gravity

  analog to: photons $\longrightarrow$ Maxwell equations

6. PLANCK VS COSMOLOGICAL SCALE

- DISCRETIZATION $\neq$ QUANTUM DISCETENESS
- GRAPH TRUNCATION = APPROXIMANTION
- QUANTUM COSMOLOGY: non-trivial interplay

7. WHAT IS THE ROLE OF QUANTUM FLUCTUATIONS?

- SUPERPOSITION: generic states of the geometry are superposed ones

TO CONCLUDE

TRUE OR FALSE?

We do not have direct access to the Plank scale. ☑

We do not have quantum-gravity measurements. ☒

We do not have quantum-gravity measurements. ☒

- Supersymmetric particles ✗
- Violation Lorentz Invariance ✗
- QG imprint on initial cosmological perturbations
- Cosmological variation of couplings
- Quantum decoherence and state collapse
- TeV Black Holes
- Generalized uncertainty principle
- Violation of discrete symmetries
- Speed of the gravitons ✗
- Gravitational Wave Echo
- Planck scale spacetime fuzziness
- …
- Entangled Masses?


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